# Event Valence and Subjective Probability*

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**Abstract**

This paper introduces the signed subjective expected utility (SSEU) model in which an individual's willingness-to-bet (WTB) on an event reflects not only the event's subjective likelihood but also its "valence"—a measure of intrinsic attractiveness or aversiveness of the event. As a result, an event's WTB may be greater than 1 or less than 0. Our model directly extends the subjective expected utility (SEU) model by weakening the Monotonicity axiom. We show that SSEU accounts for behavioral phenomena such as hedging aversion, the conjunction fallacy, coexistence of insurance and gambling, and the choice of dominated actions in strategy-proof mechanisms. We go on to show how to extend SSEU to allow for a stake-dependent (and non-additive) WTB, which also relaxes our earlier constraints on how valence can behave.

Keywords: Signed probabilities, non-monotonicity, indifference substitution, valence, attractive and aversive states

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# 1 Introduction

In the world of objective probability, where probabilities are an idealization of empirical frequencies, the probability of any event must lie between 0 and 1. This property of probability was carried over to the world of subjective probability by the pioneers, such as Ramsey (1926), de Finetti (1937), and Savage (1954), without change. Subjective probability is the mathematical representation of a person's willingness-to-bet (WTB). Under standard rationality requirements, it is a finitely additive probability, assigning to each event a weight between 0 and 1.

In this paper, we re-examine subjective probability in a world in which there can be both events with subjective probability less than 0 and events with subjective probability greater than 1; that is, we admit *signed* subjective probabilities. Choosing according to such beliefs can violate one of the basic tenets of economic rationality, namely, the Monotonicity axiom of Subjective Expected Utility (SEU), and opens the door to expressing a preference for dominated options. Our main conceptual contribution is the explanation we suggest for such dominated choices. They are due to the psychological impact that the decision maker (DM) expects from the occurrence of certain events.

The concept of *hedging aversion* provides a good starting point to understand our explanation. Consider the decision of a DM who is offered the possibility to hedge the uncertainty about the outcome of a game played by their favorite sports team. Under SEU (actually, under any model respecting Monotonicity), we would expect the DM to prefer a well-designed bet that yields a positive monetary payoff if their team loses, to offset their disappointment, to a sufficiently small negative monetary payoff in the case of a win, which partially offsets their pleasure in this case. Instead, in two lab-in-field settings, Morewedge et al. (2018) and Kossuth et al. (2020) found that sports fans preferred no bet to one that paid off in the event of a loss, even when there was no clawback in the event of a win (Morewedge et al., 2018). Donkor et al. (2023) found, in a third lab-in-field setting, that fans bet more on wins by their favorite teams as compared with other teams about whose prospects they were neutral.

In our interpretation, a DM's aversion to hedging is not based on a failure to process properly the likelihoods of events, but is explainable via an intrinsic "valence"—to use a term from psychology—on states of the world.[1] Specifically, the DM's WTB on event $E$, denoted by $\nu(E)$, results from the (additive) combination of their probabilistic belief about $E$, denoted by $p(E)$, and the valence that they associate to the event's occurrence, denoted $\gamma(E)$. Formally, we write $\nu = p + \gamma$, where $p$ is an ordinary (non-negative) probability measure and $\gamma$ is a second additive set function satisfying $\gamma(S) = 0$ and (possibly) taking positive and negative values.[2] Under conventional SEU,

[1]There is a large literature in psychology showing that individuals overweight positive events and underweight negative events (e.g., Weinstein, 1980).

[2]Note that this is not the usual Jordan decomposition of a signed measure.

a DM's belief about an event and their WTB on it coincide, so that the valence is identically 0. In contrast, our DM attributes a valence to events, with some events viewed as attractive ($\gamma(E) > 0$) and some other events viewed as aversive ($\gamma(E) < 0$).[3] We remark that Ramsey (1926) observed the possibility that events might be linked to an intrinsic desire (or the opposite) for a DM when he wrote:

> "[T]he propositions ... which are used as conditions in the options offered may be such that their truth or falsity is an object of desire to the subject. This will be found to complicate the problem, and we have to assume that there are propositions for which this is not the case, which we shall call ethically neutral."

In Ramsey's phrasing, our set-up allows for propositions (events) that are not ethically neutral, but, instead, may carry an intrinsic positive (attractive) or negative (aversive) valence. The valence function is the DM's measure of "ethical non-neutrality."

Going back to the sports betting example, if the DM assigns a negative WTB to the event that their favorite team will lose, then they will forego a hedge—even one that exhibits upside only. As mentioned, such behavior is inconsistent with SEU, and more generally with any model satisfying the well-known axiom of Monotonicity formulated by Schmeidler (1989) in his extension of the SEU axiomatization of Anscombe and Aumann (1963). This axiom states that if an act $f$ yields weakly more desirable outcomes than another act $g$, across all possible states of the world, then $f$ should be weakly preferred to $g$. In our example, let $g$ be the bet that pays off \$0 regardless of whether the DM's favorite team wins or loses, and $f$ the bet that payoffs off \$20 of their favorite team loses and \$0 otherwise. Then, under Monotonicity, the DM will strictly prefer $f$ to $g$, or, at the least, be indifferent. Yet, in the studies quoted, fans displayed a clear preference for avoiding the hedge (i.e., for bet $g$).

We replace Monotonicity with a weaker axiom called Indifference Substitution taken from Grant and Polak (2013), which requires only that if acts $f$ and $g$ yield equally desirable outcomes across all possible states of the world, then $f$ and $g$ should be indifferent. This axiom is called Substitution in Grant and Polak (2013). Our representation result shows that replacing Monotonicity by Indifference Substitution in an otherwise standard SEU model yields an additively separable representation of preferences via a Bernoulli utility $u$ on outcomes and a signed probability measure $\nu$ on states. The function $\nu$ is an additive set function that assigns measure 1 to the overall state space, but may assign measure greater than 1 or less than 0 to some events. We refer to such a representation as "signed subjective expected utility" (SSEU). Paralleling the standard model, the utility $u$ is unique up to affine transformations, and the probability $\nu$ is unique. Thus,

[3] From `https://dictionary.apa.org/valence`: "Valence [is] the subjective value of an event, object, person, or other entity in the life space of the individual. An entity that attracts the individual has positive valence whereas one that repels has negative valence."

while the SSEU representation can be seen as a type of state-dependent SEU, it separates and uniquely identifies (state-independent) utility and WTB.

We view the decomposition of the WTB $\nu$ into its (standard) probabilistic component $p$ and the valence $\gamma$, and the associated interpretation, as the conceptual contribution of this paper. However, there are arbitrarily many additive decompositions of $\nu$-. It is therefore crucial to provide motivation for the specific decomposition—we call it *canonical*—which we employ in our applications. The motivation we provide is two-fold, as we will see. First, the canonical decomposition is fully elicitable from preferences. Second, it is the decomposition that attributes to the DM the highest degree of rationality (conformity to SEU) that is compatible with their preferences.

We apply the SSEU model alongside our canonical decomposition to propose simple explanations of several well-known behavioral "anomalies." First, we return to hedging aversion (Morewedge et al., 2018; Kossuth et al., 2020; Donkor et al., 2023) and demonstrate this effect in our model. Second, we look at the conjunction fallacy (Tversky and Kahneman, 1982, 1983) from the point of view of our decomposition $\nu = p + \gamma$. We establish and interpret a sufficient condition on the ordinary probability measure $p$ and the valence $\gamma$ for the conjunction-fallacy effect to arise. Then, we turn to the classic Friedman and Savage (1948) paradox of the co-existence of insurance and gambling behavior. These authors offered a resolution that depends on changing risk attitudes with changing wealth levels. By contrast, our resolution operates at a single wealth level—so that "co-existence" becomes truly the simultaneous purchase of insurance and a lottery ticket—and it works if a risk-averse DM has a sufficiently positive valence for winning the lottery. Our final application of SSEU provides an explanation of empirical evidence that some individuals choose dominated strategies in strategy-proof mechanisms, such as the well-known deferred acceptance algorithm (Hassidim et al., 2016; Dreyfuss et al., 2022; Shorrer and Sóvágó, 2023). The key to our resolution is that the DM has a sufficiently negative valence for not being matched with their first choice.

Brandenburger et al. (2024a) is another application of signed probabilities—to an examination of the status of the Agreement Theorem (Aumann, 1976) of classical epistemics in a non-classical setting. That work was motivation for the development of the SSEU model. In their examination, Brandenburger et al. (2024a) find that conditioning becomes much more complex in a world of signed probabilities, because an event of probability 0 may contain a sub-event of strictly positive or negative probability. In a companion paper to the current one (Brandenburger et al., 2024b), we develop a theory of *signed conditional probability spaces*, extending the classical theory of conditional probability spaces due to Rényi (1955).

All the mentioned applications notwithstanding, the parsimony of SSEU naturally entails a cost in the form of a reduced fit. For example, we expect our interpretation of WTB as the sum of probabilistic beliefs and (non-zero) valence to be more tenable when the DM is evaluating bets with small monetary stakes. Also, a more complete theory of the "genesis" of a DM's WTB should take into account other established reasons for departures from the SEU benchmark, such as ambiguity attitudes (Ellsberg, 1961). For this reason, in a later section of the paper, we provide a significant extension of our model that allows for stake dependence as well as ambiguity sensitivity in the formation of WTB. This model also accommodates the experimental results in Schneider and Schonger (2019), who observed subjects jointly violating both Monotonicity and Independence, the other key axiom of the SEU model.

The organization of the rest of the paper is as follows. Section 2 lays out our decision framework, presents the SSEU representation, and introduces and discusses the decomposition of WTB into probabilistic beliefs and valence. Section 3 introduces our four axioms—Weak Order, Independence, Archimedean, and Indifference Substitution—and states our representation theorem. Section 4 presents some simple characterization and comparative statics results. Section 5 contains the explanations of hedging aversion, the conjunction-fallacy effect, the paradox of the coexistence of insurance and betting behavior, and the puzzle of choice of dominated strategies in strategy-proof mechanisms. Section 6 extends the model by weakening Independence to obtain a representation of preferences that allows for a stake-dependent WTB. Section 7 concludes with a discussion of other papers that relax Monotonicity or employ signed probabilities (e.g., Grant and Polak (2013); Dekel et al. (2001)). It also covers some conceptual matters, including the possible objection that our DM would be vulnerable to a Dutch Book (de Finetti, 1937) and the relationship of our work to complexity aversion (e.g., de Clippel et al., 2024; Puri, 2024; Gu and Chan, 2024).

# 2 Preliminaries and Basic Model

## 2.1 Choice Setting

Consider a finite[4] set $S$ of *states of the world* and a set $X$ of *consequences*. A subset $E \subseteq S$ is called an *event*. We denote by $\mathcal{F}$ the set of all functions (called *acts*) $f : S \to X$.

Given any $x \in X$, define $x \in \mathcal{F}$ to be the *constant* act such that $x(s) = x$ for all $s \in S$. With the usual slight abuse of notation, we thus identify $X$ with the subset of the constant acts in $\mathcal{F}$. If $f, g \in \mathcal{F}$, and an event $E \subseteq S$, we denote by $fEg \in \mathcal{F}$ the act that yields $f(s)$ if $s \in E$ and $g(s)$ if $s \notin E$. Given $f \in \mathcal{F}$ and $u : X \to \mathbb{R}$, $u(f)$ denotes the function $s \mapsto u(f(s))$.

We assume additionally that $X$ is a convex subset of a vector space. We can then define for

[4]To simplify the analysis, we assume a finite state space throughout. However, with appropriate modifications, all results–except those in Section 4.1—can be extended to an infinite state space. In the case of attractive and aversive states, as defined in Section 4.1, the analysis can be generalized by referring to attractive and aversive events.

every $f, g \in \mathcal{F}$ and $\alpha \in [0,1]$ the act $\alpha f + (1-\alpha)g \in \mathcal{F}$, which yields $\alpha f(s) + (1-\alpha)g(s) \in X$ for every $s \in S$. Finally, given two functions $u, u' : X \to \mathbb{R}$, we write $u \approx u'$ if there exist $a, b \in \mathbb{R}$ with $a > 0$ and $u'(x) = au(x) + b$ for every $x \in X$.

## 2.2 Signed Subjective Expected Utility

We model the decision maker's (DM's) preferences on $\mathcal{F}$ by a binary relation $\succcurlyeq$. As usual, $\succ$ and $\sim$ denote, respectively, the asymmetric and symmetric parts of $\succcurlyeq$. If $f \in \mathcal{F}$, an element $x_f \in X$ is a certainty equivalent for $f$ if $f \sim x_f$.

A signed probability measure is a function $\nu : S \to \mathbb{R}$ such that $\sum_{s \in S} \nu(s) = 1$. Given $E \subseteq S$, set $\nu(E) = \sum_{s \in E} \nu(s)$. The set $\Delta(S)$ denotes the set of all signed probability measures on $S$, that is

$$\Delta(S) = \left\{ \nu : S \to \mathbb{R} : \sum_{s \in S} \nu(s) = 1 \right\}.$$

We denote by $\Delta^+(S)$ the set of ordinary (non-negative) probability measures on $S$, i.e.

$$\Delta^+(S) = \left\{ \nu : S \to [0, \infty) : \sum_{s \in S} \nu(s) = 1 \right\}.$$

Given $\phi : S \to \mathbb{R}$ and $\nu \in \Delta(S)$, set

$$\int_S \phi d\nu := \sum_{s \in S} \phi(s)\nu(s).$$

We then have a natural extension of the SEU model to the case in which the WTB (henceforth, **WTB**) is a signed probability measure.

**Definition 1.** We say that $(u, \nu)$ is a **signed subjective expected utility** (**SSEU**) representation of $\succcurlyeq$ if there are an affine function $u : X \to \mathbb{R}$ and $\nu \in \Delta(S)$ such that

$$f \succcurlyeq g \iff \int_S u(f) d\nu \geq \int_S u(g) d\nu.$$

## 2.3 Valence Representations

We start by introducing a set function, called the *valence*, that quantifies the attractiveness or aversiveness of an event.

**Definition 2.** A function $\gamma : 2^S \to \mathbb{R}$ is called a **valence** if

1. For all $E, F \subseteq S$, if $E \cap F = \emptyset$, then $\gamma(E \cup F) = \gamma(E) + \gamma(F)$;

2. $\gamma(E) + \gamma(E^c) = 0$.

Note that Conditions 1 and 2 imply $\gamma(S) = 0$, $\gamma(\emptyset) = 0$, and $\gamma(A) = \sum_{s \in A} \gamma(s)$. Condition 1 says that the valence of an event depends on the valence of the states comprising the event (we relax this property in Section 6). Condition 2 says that if an event has a positive valence, its complement has a negative valence (again, we relax this assumption in Section 6). Moreover, the valence of the entire state space is 0.

**Definition 3.** We say that $(u, p, \gamma)$ is a **valence representation of** $\succsim$ if there are an affine function $u : X \to \mathbb{R}$, $p \in \Delta^+(S)$, and a valence function $\gamma : 2^S \to \mathbb{R}$ such that

$$f \succsim g \iff \int_S u(f)dp + \int_S u(f)d\gamma \geq \int_S u(g)dp + \int_S u(g)d\gamma.$$

If one defines the set function $\nu = p + \gamma$, the properties of the probability $p$ and of the valence $\gamma$ ensure that $\nu$ is a signed measure with $\nu(S) = 1$. Indeed, both $p$ and $\gamma$ are additive over disjoint unions and $\nu(S) = p(S) + \gamma(S) = 1$, so that $\nu \in \Delta(S)$. Therefore, any valence representation can be rewritten as an SSEU representation, as defined above.

To help in further interpreting a valence representation, consider a bet $xEy$ with $x \succsim y$. Given a valence representation $(u, p, \gamma)$ of $\nu$, its SSEU value is

$$V(xEy) = \underbrace{p(E)u(x) + (1 - p(E))u(y)}_{\text{SEU component}} + \underbrace{\gamma(E)(u(x) - u(y))}_{\text{valence component}}.$$

The value of betting on $E$ is the sum of a classical SEU component and a non-classical valence component. Since $x \succsim y$, $u(x) \geq u(y)$, so that a positive (resp. negative) valence $\gamma(E)$ will increase (resp. decrease) the overall value of the bet $xEy$. In particular, if $u(x) = 1$ and $u(y) = 0$, then $V(xEy) = p(E) + \gamma(E)$, highlighting the decomposition of the DM's WTB on $E$ into likelihood and valence components.

It is important to observe that an event commanding a WTB larger than 1 is not necessarily perceived by the DM as a "sure event," i.e., as an event with likelihood equal to 1. Clearly, the DM can assess $p(E) + \gamma(E) > 1$ while $p(E) < 1$. An analogous remark applies to events that command a negative WTB.

We now show that any SSEU representation admits a valence representation.

**Proposition 1.** *The binary relation $\succsim$ has an SSEU representation if and only if it has a valence representation.*

We sketch the proof and leave the details to Appendix A. We have already shown sufficiency. For necessity, observe that if we define $\nu^+, \nu^- \in \mathbb{R}^S$ by

$$\nu^+(s) = \max\{0, \nu(s)\} \text{ and } \nu^-(s) = \max\{0, -\nu(s)\},$$

for every $s \in S$, we obtain the Jordan decomposition

$$\nu(s) = \nu^+(s) - \nu^-(s).$$

This decomposition can in turn be written as (see Proposition 7 in Appendix A)

$$\nu^+(s) - \nu^-(s) = (1 + b)p^+(s) - bp^-(s),$$

where $p^+$, $p^- \in \Delta^+(S)$ and $b \geq 0$. A valence representation $(u, p, \gamma)$ is obtained if we let

$$p = p^+ \equiv p^* \text{ and } \gamma = b(p^+ - p^-) \equiv \gamma^*. \tag{1}$$

We call the decomposition $(p^*, \gamma^*)$ thus obtained the *canonical decomposition* of $\nu$, and the function $\gamma^*$, extended to all subsets of $S$ using the properties of a valence, the *canonical valence* of $\nu$. This terminology, and the uniqueness and observability properties of the decomposition are discussed below in Subsection 2.3.1.

**Remark 1.** *A preference relation with an SSEU representation* $(u, \nu)$ *can also be given a state-dependent representation. Indeed, define the set of "positive" states* $P \equiv \{s \in S : \nu(s) \geq 0\}$ *and of "negative" states* $N \equiv \{s \in S : \nu(s) < 0\}$.[5] *Then, the preference* $\succcurlyeq$ *has a state-dependent representation of the form*

$$V(f) = \sum_{s \in S} U(f(s), s),$$

*where*

$$U(x, s) = \begin{cases} (1 + b)u(x)p^*(s) & \text{if } s \in P, \\ u(x)(\gamma^*(s) - bp^*(s)) & \text{if } s \in N. \end{cases}$$

*This is a very special type of state dependence.*[6] *It involves only two state-dependent utilities, which differ in the sign of the marginal utility, with one positive and other other negative* ($\gamma^*(s) - bp^*(s) = -bp^-(s) \leq 0$)*, but which make the same relative comparisons between consequences. Indeed, consider two positive states* $s, s' \in P$. *The state-dependent utilities* $U(\cdot, s)$ *and* $U(\cdot, s')$ *are cardinally equivalent since they differ only by a positive constant. The same is true for any pair of negative states.*

### 2.3.1 Why "Canonical"?

The terminology introduced above reflects the view that, while arbitrarily many decompositions of the signed measure $\nu$ are possible, the decomposition $(p^*, \gamma^*)$ arises naturally. Here we substantiate this claim by showing that $(p^*, \gamma^*)$ distinguishes itself because of its *observability* (from preferences) and because of its *parsimony* (as a departure from SEU).

[5]The sets $P$ and $N$ can be derived from preferences. It is straightforward to show that $P = \{s \in S : x\{s\}y \succcurlyeq y \text{ for some } x \succ y\}$ and $N = \{s \in S : y \succ x\{s\}y \text{ for some } x \succ y\}$.

[6]The identification of probabilistic beliefs for general state-dependent utility preferences requires observing many conditional preferences (Karni et al., 1983; Lu, 2019). As we show below, the identification of $p^*$ (and $\gamma^*$) only requires observing $\succcurlyeq$.

Indeed, consider the sets $P$ and $N$ defined in Remark 1. The probability $p^*$ is the normalized restriction of the signed measure $\nu$ to the set $P$, that is: $p^*(s) = \frac{\nu(s)}{\nu(P)}$ for $s \in P$ and $p^*(s) = 0$ otherwise. Since $\nu$ is uniquely identified from $\succsim$, and $P$ is also uniquely identified (up to $\nu$-measure zero events), it follows that $p^*$ is uniquely identified. Given knowledge of the SSEU representation $(u, \nu)$,[7] it is also behaviorally observable: Fix $x, y \in X$ with $x \succ y$. Then, for every $s \in P$, we can obtain $p^*(s)$ by eliciting the indifference $x\,\{s\}\,y \sim z$ for some $z \in X$, since, after normalizing $u$ so that $u(x) = \nu(P)$ and $u(y) = 0$, we have $V(x\,\{s\}\,y) = \nu(P)\nu(s) = p^*(s) = u(z)$. Given $p^*$, the canonical valence is then just $\gamma^* = \nu - p^*$.

The choice of $p^*$ can also be given a more conceptual justification in terms of modeling strategy. Following Savage (1954), we can define a preference $\succsim_P$ on $\mathcal{F}$ as follows: for every $f, g \in \mathcal{F}$

$$f \succsim_P g \iff f\,P\,h \succsim g\,P\,h \text{ for some } h \in \mathcal{F},$$

(where the definition does not depend on $h$ since $\succsim$ satisfies Independence). One can interpret $\succsim_P$ as a preference conditional on the DM being informed that event $P$ obtained, or, as in Savage's treatment, just as a hypothetical preference (preference "given $P$"). We have the following result:

**Proposition 2.** *Suppose that the binary relation $\succsim$ is an SSEU preference with (canonical) valence representation $(u, p^*, \gamma^*)$. Then $\succsim_P$ has an SEU representation $(u, p^*)$.*

So, if the SSEU preference has a valence representation with canonical probability $p^*$, the same probability will be used to represent the "given $P$" preference, which is an SEU preference. When evaluating acts that behave identically outside of $P$, the DM behaves as if they had the standard probabilistic beliefs $p^*$ (which are unique by the SEU representation result). This provides the second justification for choosing $p^*$ in the "canonical" decomposition. It is the probability that the (SEU-rational) DM would use if they could restrict the domain to the positive states (for example, in a conditional preference setting with consequentialism). Of course, it is possible that the DM might also attach valence to some states $s \in P$, but in choosing $p^*$ we are following a "best rationalization"[8] modeling strategy of trying to depart as little as possible from the SEU "rationality" benchmark when representing the DM's preferences.

We close the discussion in this section with two interesting properties of $(p^*, \gamma^*)$ that stem from the previous observations. First, $(p^*, \gamma^*)$ minimizes the overall "impact" of $\gamma$, thus providing the "most classical" description of the DM's preferences. For any $\phi : S \to \mathbb{R}$, let $\|\phi\|_v$ denote the total variation of $\phi$, i.e., $\|\phi\|_v = \sum_{s \in S} |\phi(s)|$. We have:[9]

**Proposition 3.** *Let $\nu \in \Delta(S)$ be a signed probability measure and $(p^*, \gamma^*)$ be its canonical decomposition. Then*

$$p^* \in \operatorname*{argmin}_{q \in \Delta^+(S)} \|\nu - q\|_v \,. \tag{2}$$

[7] That is, given that we have observed the entire preference relation $\succsim$.

[8] This term originates in game theory; see Battigalli (1996).

[9] We owe the proof of this result to Luigi Montrucchio.

It follows from Eq. (2) that for any alternative decomposition $(p, \gamma)$ the canonical decomposition $(p^*, \gamma^*)$ satisfies

$$\sup_{A \subseteq S} |\gamma^*(A)| \leq \sup_{A \subseteq S} |\gamma(A)|.$$

Second, as a direct consequence of its construction, the pair $(p^*, \gamma^*)$ preserves the likelihood ratios for states with positive WTB: Suppose that the DM observes an event $E \subseteq P$. Then, using the definition of conditional probability (whenever it is defined), we obtain

$$\frac{\nu(s|E)}{\nu(s'|E)} = \frac{\nu(s)}{\nu(s')} = \frac{p^*(s)}{p^*(s')} = \frac{p^*(s|E)}{p^*(s'|E)},$$

for every $s, s' \in E$. By contrast, for any other decomposition $(p, \gamma)$, we will have $\nu(s|E)/\nu(s'|E) \neq p(s|E)/p(s'|E)$ for some $s, s'$. Thus, the likelihood $p^*$ chosen via our canonical decomposition is the unique one that is consistent with Bayesian updating of $\nu$. The importance of this updating property of the canonical decomposition is explored further in Brandenburger et al. (2024b).

# 3 Axiomatic Characterization

This section introduces the axioms characterizing our SSEU model. Start with the standard Anscombe-Aumann axioms that characterize SEU.[10]

**Axiom 1** (Weak Order - WO)**.** $\succsim$ is complete and transitive. Moreover, there exist $f, g \in \mathcal{F}$ such that $f \succ g$.

**Axiom 2** (Independence - I)**.** If $f, g, h \in \mathcal{F}$ and $\gamma \in (0, 1]$, $f \succsim g$ implies $\gamma f + (1 - \gamma)h \succsim \gamma g + (1 - \gamma)h$.

**Axiom 3** (Archimedean - A)**.** If $f, g, h \in \mathcal{F}$ and $f \succ g \succ h$, there are $\alpha, \beta \in (0, 1)$ such that $\alpha f + (1 - \alpha)h \succ g \succ \beta f + (1 - \beta)h$.

**Axiom 4** (Monotonicity - M)**.** For every $f, g \in \mathcal{F}$, $f(s) \succsim g(s)$ for every $s \in S \implies f \succsim g$.

Recalling the discussion in the Introduction, we weaken Monotonicity as follows[11]

**Axiom 5** (Indifference Substitution - IS)**.** For every $f, g \in \mathcal{F}$, $f(s) \sim g(s)$ for every $s \in S \implies f \sim g$.

[10] The independence axiom could be weakened, following Principi et al. (2023).

[11] One potential explanation for violation of Monotonicity is that this entails a subtle form of "state independence" of preferences, or weak separability, which may be overly restrictive. Monotonicity implies that, if $xEf \succ yEf$ for some event $E$ and some act $f$, then $xFg \succsim yFg$ for all events $F$ and acts $g$. In particular, if $x \succsim y$ then $xEf \succsim yEf$ for all events $E$ and all acts $f$.

The IS axiom appears in Grant and Polak (2013), where it is called Substitution. Under IS, two acts that yield equivalent payoffs in all states must be deemed indifferent. To illustrate the weaker scope of IS compared with Monotonicity, consider two payoffs $x \succ y$. Under Monotonicity, it is necessary that $x \succcurlyeq xEy$ for all events $E$. However, IS allows for the possibility of $xEy \succ x$, that is, that the event $E$ is "attractive" (see Section 4.1).

Replacing M with IS characterizes our SSEU representation.

**Theorem 1.** *The binary relation $\succcurlyeq$ satisfies axioms WO, I, A, and IS if and only if there exists a non-constant affine function $u : X \to \mathbb{R}$ and a signed probability measure $\nu \in \Delta(S)$ such that $(u, \nu)$ is a SSEU representation of $\succcurlyeq$. Moreover, if $(u', \nu')$ is another SSEU representation of $\succcurlyeq$, then $u \approx u'$ and $\nu' = \nu$.*

By Proposition 1, the axioms in Theorem 1 are necessary and sufficient to obtain a valence representation of $\succcurlyeq$.

The next result shows that, in the context of SSEU preferences, imposing M in place of IS is tantamount to assuming the existence of a valence representation of $\succcurlyeq$ in which all events have "zero" valence (or, equivalently, assuming that all states are positive).

**Corollary 1.** *Suppose that a binary relation $\succcurlyeq$ satisfies axioms WO, I, A, and IS. Then the following are equivalent:*

*(i)* $\succcurlyeq$ *satisfies axiom M,*

*(ii)* $\succcurlyeq$ *has a valence representation $(u, p, \gamma)$ where $\gamma(E) = 0$ for all $E \subseteq S$,*

*(iii)* $P = S$.

To provide additional perspective, we present a few examples of SSEU preferences. In the first two cases, we employ canonical decompositions. The other two cases employ decompositions that are intuitive but not canonical.

**Example 1.** Suppose there is only one negative state $s^*$. That is, $N = \{s^*\}$, so that $\gamma^*(s) = b(p^*(s) - \delta_{s^*})$. Then

$$V(f) = \int_S u(f)dp^*(s) + b\left(\int_S u(f)dp^*(s) - u(f(s^*))\right).$$

Consider the bet $xEy$ for some event $E$ with $s^* \in E^c$. Then

$$V(xEy) = p^*(E)u(x) + (1 - p^*(E))u(y) + bp^*(E)(u(x) - u(y)),$$

and the WTB on $E$ is $\nu(E) = (1 + b)p^+(E)$. For general acts, if $f \succcurlyeq g$, then

$$\int_S u(f)dp^* - \int_S u(g)dp^* \geq \frac{b}{1+b}(u(f(s^*)) - u(g(s^*)).$$

Suppose $u(f(s^*)) - u(g(s^*)) \geq 0$. Then a preference for $f$ over $g$ implies that the SEU component of the value of $f$ is sufficiently larger than that of $g$ to override the negative value of obtaining more utility in the state $s^*$. If $u(f(s^*)) - u(g(s^*)) \leq 0$, a preference for $f$ over $g$ can obtain even if the SEU component of the value of $g$ is strictly larger than that of $f$. △

**Example 2.** Paralleling Example 1, suppose that there is only one positive state $s^*$. That is, $P = \{s^*\}$, so that $\gamma^*(s) = b(\delta_{s^*} - p^-(s))$. Then

$$V(f) = u(f(s^*)) + b\left(u(f(s^*)) - \int_S u(f(s))dp^-\right).$$

Consider the bet $xEy$ for some $E$ with $s^* \in E$. Then

$$V(xEy) = u(x) + b(1 - p^-(E))(u(x) - u(y)),$$

and the WTB on $E$ is $\nu(E) = 1 + b(1 - p^-(E))$. For general acts, if $f \succcurlyeq g$, then

$$u(f(s^*)) - u(g(s^*)) \geq \frac{b}{1+b}\left(\int_S u(f)dp^- - \int_S u(g)dp^-\right).$$

△

**Example 3.** Consider an SSEU preference with a (non-canonical) valence given by

$$\gamma(s) = k(p(s) - 1/|S|),$$

for some $k \geq 0$ and $p \in \Delta^+(S)$. Then

$$V(f) = \int_S u(f)dp(s) + k\left(\sum_{s\in S}(p(s)u(f(s)) - \frac{1}{|S|}\right).$$

There is positive (resp. negative) valence if a state is more (resp. less) likely than the uniform case. The WTB on an event $E$ is $\nu(E) = p(E) + k\,(p(E) - |E|/|S|)$. △

**Example 4.** In the spirit of the Radon-Nikodym derivative, consider a (non-canonical) valence given by $\gamma(s) = k\phi(s)p(s)$ for some $k \geq 0$ and a distortion function $\phi : S \to \mathbb{R}$ with $\int_S \phi dp = 0$ (in the spirit of the Vector Expected Utility of Siniscalchi, 2009). Then

$$V(f) = \int_S u(f)dp + \int_S u(f)\phi dp.$$

The WTB on event $E$ is $\nu(E) = p(E) + \int_E \phi dp$. △

## 4 Some Characterizations and Comparative Statics

In this section we characterize some intuitive concepts within the SSEU framework.

## 4.1 Attractive and Aversive States

The DM in our model attributes a valence to states, with some states viewed as attractive and other states as aversive. These notions can be characterized formally in terms of the canonical valence.

**Definition 4.** Fix a preference relation $\succcurlyeq$ with an SSEU representation $(u, \nu)$, where $u$ is non-constant, and let $(p^*, \gamma^*)$ be the canonical decomposition of $\nu$. A state $s \in S$ is **attractive** (resp. **aversive**) if $\gamma^*(s) > 0$ (resp. $\gamma(s)^* < 0$).

A notable feature of our canonical valence concept is that it yields a simple behavioral characterization of aversive states, but only a sufficient condition for a state to be attractive.

**Proposition 4.** *Fix a preference relation $\succcurlyeq$ with an SSEU representation $(u, \nu)$, where $u$ is non-constant, and let $(p^*, \gamma^*)$ be the canonical decomposition of $\nu$. A state $s \in S$ is aversive if and only if $y \succ x\{s\}y$ for some $x \succ y$. A state $s$ is attractive if $x\{s\}y \succ y$ for some $x \succ y$.*

We see that aversive states are exactly those states on which the DM would not bet regardless of the payoffs. By contrast, if a state is such that the DM prefers to bet on it regardless of the payoffs, then that state is attractive.

## 4.2 Null Events

When a DM entertains signed probabilities, an event can be null but contain states that have non-zero probability. It therefore makes sense to posit that updating should in some way preserve beliefs over these internal states. We consider in particular the following distinct notions of null and non-null event.[12]

**Definition 5.** An event $E \subseteq S$ is $\succcurlyeq$**-null** if $xEy \sim y$ for some $x \not\sim y$; it is $\succcurlyeq$**-non-null** otherwise. An event $E \subseteq S$ is $\succcurlyeq$**-completely null** if every $F \subseteq E$ is $\succcurlyeq$-null. An event $E \subseteq S$ is $\succcurlyeq$**-classically null** if there is no $F \subseteq E$ such that $xFy \succ y$ for some $x \succ y$.

The preceding definitions above coincide under SEU, but differ under SSEU. This can be seen from the following characterizations in terms of the canonical decomposition $(p^*, \gamma^*)$.

**Proposition 5.** *Fix a preference relation $\succcurlyeq$ with an SSEU representation $(u, \nu)$ where $u$ is non-constant, and let $(p^*, \gamma^*)$ be the canonical decomposition of $\nu$. An event $E$ is $\succcurlyeq$-null if and only if $p^*(E) + \gamma^*(E) = 0$. An event is $E$ $\succcurlyeq$-completely null if and only if $p^*(F) + \gamma^*(F) = 0$ for all $F \subseteq E$. An event $E$ is $\succcurlyeq$-classically null if and only if $p^*(E) = 0$.*

[12] We are grateful to Miklós Pintér for suggesting the definition of a completely null event.

## 4.3 Comparative Statics of Valence

The absolute value of the valence function can be used as a measure to quantify the degree of non-classicality of a signed probability $\nu$. A larger value of $|\gamma^*|$ corresponds to a more non-classical probability.

Consider two DM's with preferences $\succcurlyeq_1$ and $\succcurlyeq_2$. We show how the DM's can be ranked in terms of the extent to which their preferences depart from the classical framework. For $i = 1, 2$, let $P_i$ and $N_i$ be the sets defined in Remark 1 (also Footnote 5).

**Definition 6.** Say that DM 1 is **more non-classical** than DM 2 if, for all $f, g, h \in \mathcal{F}$ and $E \subseteq P_1 \cup P_2$,

$$fEh \succcurlyeq_1 gEh \implies fEh \succcurlyeq_2 gEh, \tag{3}$$

$$fEh \succ_1 gEh \implies fEh \succ_2 gEh, \tag{4}$$

and for every $x, y, z \in X$ with $x \succ y$ and $E \subseteq N_1 \cup N_2$,

$$xEy \sim_1 z \implies xEy \succcurlyeq_2 z. \tag{5}$$

To understand this notion, observe that conditions (3) and (4) imply that, conditional upon the realization of a positive event, the two DMs rank bets in the same way. It follows that if one positive event is considered more likely than another positive event according to DM 1, then this will also be the case for DM 2. The same holds for strict ranking. Furthermore, condition (5) indicates that DM 1 demands a higher compensation than DM 2 to participate in a bet involving a negative event. The next result shows how the absolute value of the valence function is a measure of non-classicality, which further motivates why our decomposition provides the "most classical" description of a DM's behavior.

**Theorem 2.** *Given DMs 1 and 2 with preferences admitting an SSEU representation, the following are equivalent:*

*(i) DM* 1 *is more non-classical than DM* 2*;*

*(ii)* $\succcurlyeq_1$ *and* $\succcurlyeq_2$ *admit respective SSEU representations* $(u_1, \nu_1)$ *and* $(u_2, \nu_2)$ *with canonical decompositions* $\nu_1 = (p_1^*, \gamma_1^*)$ *and* $\nu_2 = (p_2^*, \gamma_2^*)$ *where* $u_1 = u_2$, $p_1^* = p_2^*$, *and* $|\gamma_1^*| \geq |\gamma_2^*|$.

In addition, if DM 1 is more non-classical than DM 2, so that $|\gamma_1^*| \geq |\gamma_2^*|$, then $b_1 \geq b_2$. To see this, use $|b_1(p_1^+(s) - p_1^-(s))| \geq |b_2(p_2^+(s) - p_2^-(s))|$ for all $s \in S$, where $p_1 = p_2 = p$. Choosing a positive state $s$ yields $|b_1 p^+(s)| \geq |b_2 p^+(s)|$, or $b_1 \geq b_2$.

In the next example we consider simple parametric specifications of non-classicality.

**Example 5** (Continuing Examples 1 and 3)**.** Suppose that DM 1 and DM 2 each have one negative state $s_1^*$ and $s_2^*$, respectively. Thus, $\gamma_1(s) = b_1(p_1^+(s) - \delta_{s_1^*})$ and $\gamma_2(s) = b_2(p_2^+(s) - \delta_{s_1^*})$.

If DM 1 is more non-classical than DM 2, then $p_1 = p_2 = p$, $u_1 = u_2$, and $|\gamma_1^*| \geq |\gamma_2^*|$. It follows that $s_1^* = s_2^* = s^*$. Using again $|\gamma_1^*| \geq |\gamma_2^*|$, we conclude that $|b_1(p(s) - \delta_{s^*})| \geq |b_2(p(s) - \delta_{s^*})|$, which is equivalent to $b_1 \geq b_2$.

Next, consider $\gamma(s) = b(p^+(s) - \frac{1}{|S|})$, so that

$$V(f) = \int_S u(f)dp^+ - b\left(\sum_{s\in S}(p^+(s) - \frac{1}{|S|})u(f(s))\right).$$

Again, we see that the degree of non-classicality is parameterized by $b$, where a larger $b$ indicates a larger departure from classicality. △

# 5 Applications

In this section, we provide some simple examples of the potential of event valence in explaining behavior that is hard to reconcile with classical SEU. Unless otherwise noted, throughout the section we assume that the DM has SSEU preferences $\succcurlyeq$ with some valence representation $(u, p, \gamma)$.

## 5.1 Hedging Aversion

Experimental evidence indicates reluctance on the part of sports fans to bet against a win by their favorite team (Morewedge et al., 2018; Kossuth et al., 2020; Donkor et al., 2023). This goes against standard SEU theory, which predicts that a well-designed hedge can mitigate disappointment generated by unfavorable outcomes.

In Morewedge et al. (2018), supporters of a sports team preferred a sure payment of \$0 to a bet paying \$20 if their preferred team lost a certain game. This preference was typically reversed when the bet was on a loss by a team different from the bettor's preferred one – even when the two teams had the same likelihood of losing. Rejecting a bet paying \$20 in favor of a sure payment of \$0 is a direct violation of Monotonicity (under the natural assumption that \$20 is preferred to \$0). Also, a difference in WTB on two equally likely events is inconsistent with standard SEU theory, under which the likelihood of an event and the WTB on it coincide.

To see that our SSEU theory can accommodate this pattern of behavior, let $E$ be the event in which the DM's favorite team loses. Then $V(\$0) > V(\$20E\$0)$ whenever

$$u(0) > p(E)u(20) + (1 - p(E))u(0) + \gamma(E)(u(20) - u(0)),$$

which holds if and only

$$p(E) + \gamma(E) < 0.$$

Intuitively, if the event $E$ has a sufficiently negative valence, the DM's WTB on $E$ becomes negative.

Now let $F$ be the event that a team different from the DM's preferred one loses a game. Again, the DM chooses between a sure payment of \$0 and a bet paying \$20 if this team loses. Even if the two events $E$ and $F$ have the same likelihood, i.e., $p(E) = p(F)$, SSEU theory is consistent with both $V(\$0) > V(\$20E\$0)$ and $V(\$20F\$0) > V(\$0)$ whenever

$$-\gamma(E) > p(E) = p(F) > \gamma(F).$$

A particular case is $\gamma(F) = 0$, which says that the event $F$ has no valence for the DM. Under SEU, we also have $\gamma(E) = 0$, so that this preference pattern is impossible.[13]

## 5.2 Conjunction Fallacy

Individuals exhibit the conjunction fallacy when they consider the probability of a conjunction of two events to be higher than the probability of one of the constituent events (Tversky and Kahneman, 1982, 1983). Our SSEU model can accommodate the conjunction fallacy, as the following immediate result illustrates:

**Fact 1.** *For any* $E, F \subseteq S$, *if* $\gamma(E \cap F) - \gamma(F) = -\gamma(F \backslash E) > p(F) - p(E \cap F)$, *then* $\nu(E \cap F) > \nu(F)$.

For example, consider the following "update" of a famous experiment by Tversky and Kahneman (1983).[14] Conser the following four events concerning a tennis match played by Italian star Jannik Sinner (in parentheses, their depiction in Table 1):

- $E_1$ = Sinner wins the match (rounded solid box)
- $E_2$ = Sinner loses the first set (solid box)
- $E_3$ = Sinner loses the first set but wins the match (dotted box)
- $E_4$ = Sinner wins the first set but loses the match (dashed box)

In the Tversky-Kahneman experiment, subjects (on average) ranked event $E_1$ more probable than event $E_3$, event $E_3$ more probable than $E_2$, and event $E_4$ as the least probable. A conjunction fallacy arises because event $E_3$ is the conjunction of events $E_1$ and $E_2$; that is, $E_3 = E_1 \cap E_2$. The probability of $E_3$ should not be larger than the probability of $E_2$.

Under our SSEU theory, a DM will exhibit the conjunction fallacy if $\nu(E_3) > \nu(E_2)$. This will happen if the valence of the state $s_2$, where Sinner loses both the first set and then the match, is sufficiently negative. For a Sinner fan, satisfaction of this condition could reflect the (anticipated) disappointment after a "total" loss.

[13] Morewedge et al. (2018) found that as the financial gain increased (from \$20), fans were more likely to accept the bet against their team. The generalization of our model laid out in Section 6 can accommodate this preference pattern.

[14] The events in their experiment involved tennis great Björn Borg.

| First set \ Match | win | lose |
|---|---|---|
| win | $s_1$ | $s_4$ |
| lose | $s_3$ | $s_2$ |

Table 1: Events in the Conjunction Fallacy

**Fact 2.** *Suppose that* $p(s_1) > -\gamma(s_1)$*,* $p(s_2) < -\gamma(s_2)$*, and* $p(s_4) + \gamma(s_4)$ *is sufficiently small. Then*

$$\nu(E_1) > \nu(E_3) > \nu(E_2) > \nu(E_4).$$

To see this, observe that by the first condition, $\nu(E_1) = p(s_1)+p(s_3)+\gamma(s_1)+\gamma(s_3) > p(s_3)+\gamma(s_3) = \nu(E_3)$. By the second condition, $\nu(E_3) = p(s_3) + \gamma(s_3) > p(s_3) + p(s_2) + \gamma(s_3) + \gamma(s_2) = \nu(E_2)$. Lastly, if $p(s_4) + \gamma(s_4) < \nu(E_2)$, we obtain the desired ranking.

As an example, the valence function defined by $\gamma(s_1) = -\gamma(s_2) > p(s_2)$ and $\gamma(s_3) = \gamma(s_4) = 0$ satisfies the conditions in Fact 2.

If, in addition to the conditions of Fact 2, we have $\gamma(E_1) = \gamma(E_1 \cap E_2) = \gamma(E_3)$, then $\nu(E_1) > \nu(E_1 \cap E_2) = \nu(E_3) > \nu(E_2)$. Next, consider the valence defined in Example 2, viz., $\gamma(s) = b(\delta_{s^*} - p^-(s))$. Let $s_1 = E_1 \backslash E_2$ be the attractive state. Then

$$\begin{aligned}
\nu(E_1) &= 1 + b(1 - p^+(E_1)), \\
\nu(E_2) &= -bp^-(E_2), \\
\nu(E_3) &= \nu(E_1 \cap E_2) = -bp^-(E_1 \cap E_2).
\end{aligned}$$

Thus, we find $\nu(E_1) > \nu(E_1 \cap E_2) = \nu(E_3) > \nu(E_2)$. If $p(s_4) = \nu(E_4)$ is smaller than $\nu(E_2)$, we obtain the desired ranking.

## 5.3 Coexistence of Insurance and Gambling

Standard SEU with a concave or convex utility function struggles to explain why individuals simultaneously buy insurance and lottery tickets. The first purchase reflects risk aversion, while the second reflects risk seeking. Friedman and Savage (1948) suggests that risk attitudes can vary with wealth levels, so that insurance and gambling can coexist in "sequential" purchases. We use the SSEU model to propose an alternative explanation, based on valence, that does not require sequencing of purchases.[15]

[15] If the price of a lottery ticket is small enough, the sequential explanation would require a large local change in the curvature of the utility function.

An insurance policy covers a loss $\ell \geq 0$ if event $L$ occurs. The premium is denoted by $\psi$, and we assume that $0 \leq \nu(L) \leq 1$ and that $\psi = \nu(L)\ell$. A lottery pays $w \geq 0$ if the event $E$ occurs and 0 otherwise. The price $\pi$ of the lottery ticket is $0 \leq \pi \leq w$. The DM's current wealth is $x$.

| | $L$ | $L^c$ |
|---|---|---|
| $f_1$ | $x - \ell$ | $x$ |
| $f_2$ | $x - \psi$ | $x - \psi$ |

Table 2: Acts $f_1$ and $f_2$

The DM compares four options $\mathcal{F} = \{f_1, f_2, f_3, f_4\}$. The first option $f_1$ is to choose to purchase neither the insurance nor the lottery ticket. The first row of Table 2 shows the associated payoffs. The SSEU is

$$V(f_1) = (p(L) + \gamma(L))u(x - \ell) + (1 - p(L) - \gamma(L))u(x).$$

The second option $f_2$ is to buy the insurance but not the lottery ticket. The second row of Table 2 shows the associated payoffs. The SSEU is

$$V(f_2) = u(x - \psi).$$

The third option $f_3$ is to buy the lottery ticket but not the insurance. Table 3 shows the associated payoffs. The SSEU is

$$V(f_3) = (p(L) + \gamma(L)) \left[(p(E) + \gamma(E))u(x - \pi + w - \ell) + (1 - p(E) - \gamma(E))u(x - \pi - \ell)\right] + (1 - p(L) - \gamma(L)) \left[(p(E) + \gamma(E))u(x - \pi + w) + (1 - p(E) - \gamma(E))u(x - \pi)\right].$$

Finally, with option $f_4$ the individual purchases both the insurance and the lottery ticket. Table 4 shows the associated payoffs. The SSEU is

$$V(f_4) = (p(E) + \gamma(E))u(x - \pi + w - \psi) + (1 - p(E) - \gamma(E))u(x - \pi - \psi).$$

| | $L$ | $L^c$ |
|---|---|---|
| $E$ | $x - \pi + w - \ell$ | $x - \pi + w$ |
| $E^c$ | $x - \pi - \ell$ | $x - \pi$ |

Table 3: Act $f_3$

The coexistence of gambling and insurance requires $f_4 \succcurlyeq f_1$.

**Fact 3.** *Assume that the utility $u$ is increasing and concave. If winning the lottery has a positive valence such that $p(E) + \gamma(E) \geq 1$, then there is coexistence of gambling and insurance.*

| | $L$ | $L^c$ |
|---|---|---|
| $E$ | $x - \pi + w - \psi$ | $x - \pi + w - \psi$ |
| $E^c$ | $x - \pi - \psi$ | $x - \pi - \psi$ |

Table 4: Act $f_4$

Actually, the conditions of Fact 3 imply $f_4 \succcurlyeq f_2 \succcurlyeq f_1$. Moreover, since $u$ is concave, ceteris paribus, the DM always buys insurance, thus $f_4 \succcurlyeq f_3$. We see that if utility is concave and winning the lottery is a sufficiently attractive event—that is, if $p(E) + \gamma(E) > 1$—an SSEU individual will buy both the insurance policy and the lottery ticket. Note that Fact 3 is true at any given level of wealth $x$: our analysis does not depend on wealth effects. Instead, our mechanism operates by using the DM's valence to offset risk aversion. A sufficiently high valence creates risk-seeking behavior even for a DM with concave utility function $u$. The condition can be stated in terms of what is known as Jensen's gap (for the probabilistic part $p$ of the valence representation of $\nu$)

$$u[p(E)x + (1 - p(E))y] - [p(E)u(x) + (1 - p(E))u(y)].$$

If this gap is less than the quantity $[u(x) - u(y)]\gamma(E)$, then, despite the concavity of $u$, the DM will choose the bet between $x$ and $y$ over its $p$-expectation $p(E)x + (1 - p(E))y$.

**Fact 4.** *For any bet $xEy$ with $x \succcurlyeq y$, if $u$ is increasing and $p(E) + \gamma(E) \geq 1$, then $V(xEy) \geq u(p(E)x + (1 - p(E))y)$.*

The condition $p(E)+\gamma(E) \geq 1$ implies $\gamma(E) \geq 0$. Thus, if $u$ is convex, the preceding inequality is trivially satisfied since the right-hand side is negative and the left-hand side is positive. The interesting case is when $u$ is concave, and yet a "risk averse" DM can prefer a lottery to its expected value.

To link Fact 4 to the coexistence of gambling and insurance, note that if $p(E) + \gamma(E) \geq 1$, then, by Fact 3, $V(f_4) \geq u(x - \psi)$, but $u(x - \psi) \geq u(x - \psi - \pi + w) \geq u(x - \psi - \pi + p(E)w)$. Therefore, $V(f_4) \geq u(x - \psi - \pi + p(E)w)$, and $(x - \psi - \pi + p(E)w)$ is the $p$-expected value of the lottery in the presence of insurance.

## 5.4 Dominated Choice in Strategy-Proof Mechanisms

Recent empirical evidence suggests that some individuals choose dominated (in the sense of first-order stochastic dominance) strategies in strategy-proof mechanisms, such as the Deferred Acceptance (DA) mechanism much studied in the context of school or college choice (Hassidim et al., 2016; Shorrer and Sóvágó, 2023; Dreyfuss et al., 2022).

Flipping and truncation are two examples of dominated strategy choice. Under flipping, an individual submits a ranking that reverses the "obvious" order of two alternatives—e.g., ranking a school choice that comes with a scholarship below the same choice without a scholarship. Under truncation, an individual submits a restricted ranking that omits some schools. Both types of behavior are inconsistent with standard SEU, which respects first-order stochastic dominance. Instead, they are consistent with our SSEU theory.

Consider two schools $\sigma_1$ and $\sigma_2$. Let $u(x_1) = m_1 > 0$ denote the utility of being matched with school $\sigma_1$, $u(x_2) = m_2$ the utility of being matched with school $\sigma_2$, and $u(x_0) = 0$ the utility of not being matched. Clearly, being matched with both $\sigma_1$ and $\sigma_2$ is not possible under the DA. There are four possible rankings, which we denote by $\sigma_1 \triangleright \sigma_2$, $\sigma_2 \triangleright \sigma_1$, $\sigma_1$, and $\sigma_2$. The first two rankings are complete, while the third and fourth are truncations.

Let $E_1$ be the event of being matched with school $\sigma_1$, $E_2$ the event of being matched with school $\sigma_2$, and $E_{\neg 1,2}$ the event of being matched with school $\sigma_2$ conditional on not being matched with school $\sigma_1$. Submitting the ranking $\sigma_1 \triangleright \sigma_2$ generates the act that yields $x_1$ if $E_1$ occurs, $x_2$ if $E_{\neg 1,2}$ occurs, and $x_0$ otherwise. An analogous act corresponds to submitting $\sigma_2 \triangleright \sigma_1$. Following Dreyfuss et al. (2022), we assume for simplicity that the probability assessment of being matched to school $\sigma_2$ conditional on not being matched to school $\sigma_1$ can be written as $p(E_{\neg 1,2}) = (1 - p(E_1))p(E_2)$, and similarly for the event of being matched to $\sigma_1$ conditional on not being matched to $\sigma_2$.

Submitting a ranking that contains only $\sigma_1$ generates the act $x_1 E_1 x_0$ that yields $x_1$ if $E_1$ occurs and $x_0$ otherwise. Likewise, submitting a ranking that contains only $\sigma_2$ generates the act $x_2 E_2 x_0$ that yields $x_1$ if $E_2$ occurs and $x_0$ otherwise.

Suppose that an individual ranks school $\sigma_1$ higher than $\sigma_2$, that is, $m_1 > m_2$. Flipping means that the individual submits the ranking $\sigma_2 \triangleright \sigma_1$. The SSEU of submitting a faithful ranking is $\nu(E_1)m_1 + \nu(E_{\neg 1,2})m_2$, whereas the SSEU of the flipped ranking is $\nu(E_2)m_2 + \nu(E_{\neg 2,1})m_1$. Submitting a flipped ranking is preferred if

$$p(E_1)p(E_2)(m_1 - m_2) < (\gamma(E_{\neg 2,1}) - \gamma(E_1))m_1 - (\gamma(E_{\neg 1,2}) - \gamma(E_2))m_2.$$

The left-hand side of this inequality is always positive, and so the right-hand side has to be sufficiently positive if flipping is to be optimal. For example, suppose that all events in the inequality have valence 0 except for the aversive event $E_{\neg 1,2}$, corresponding to being matched with the less preferred school $\sigma_2$ over the more preferred school $\sigma_1$. The inequality then becomes

$$p(E_1)p(E_2)(m_1 - m_2) < -m_2\gamma(E_{\neg 1,2}),$$

which will hold if $\gamma(E_{\neg 1,2})$ is sufficiently smaller than 0. By contrast, since $\gamma = 0$ under SEU, this inequality can never be satisfied.

Truncation, by submitting just $\sigma_1$ rather than the ranking $\sigma_1 \triangleright \sigma_2$, is preferred if

$$\nu(E_1)m_1 > \nu(E_1)m_1 + \nu(E_{\neg 1,2})m_2,$$

which will hold whenever $\gamma(E_{\neg 1,2}) < 0$ is small enough, which will hold if there is sufficient aversion to being rejected by the more preferred school.

Our explanation of flipping and truncation behavior in the DA mechanism differs from that of Dreyfuss et al. (2022). These authors suggest that "an applicant who is likely to get matched with a school will feel a loss when matched with any other school (even a better one); this can create attachment to the high-probability school—an endowment effect for schools." From this idea they conclude that flipping and even truncation might be observed in highly loss-averse individuals. Our explanation is not dependent on a (high) likelihood of being matched with a particular school, but on the aversiveness of not being matched with a preferred school.

# 6 Stake Dependence (and Other Generalizations)

Given an SSEU preference represented by $(u, \nu)$ and an aversive state $s$, a DM will not be willing to buy any bet $x\{s\}y$ regardless of the size of the stakes $x$ and $y$. This is a consequence of the fact that SSEU preferences satisfy Independence, which yields the separable structure $V(x\{s\}y) = u(x)\nu(s) + u(y)(1 - \nu(s))$. While this "stake independence" is a feature that is shared by the classical SEU model and by most of its well-known generalizations,[16] it strikes us as particularly implausible when describing DMs who may attach valence to some states or events. A supporter who rejects a bet on their preferred sports team losing when the payoff is small, might well accept the same bet for a sufficiently large payoff. Therefore, in this section we show how to modify our analysis to allow a DM to have a WTB—and hence valence—that is stake-dependent. The model we develop is very general. Thus, our substantial relaxation of Independence makes it possible to incorporate other factors affecting WTB for a DM. For example, we can admit ambiguity attitudes (Ellsberg, 1961). A cost of this additional generality is that the DM's valence function loses its simple additivity property.

As announced, admitting stake dependence requires relaxing the Independence axiom. We replace it with the following very weak version, which requires independence only in the comparison of constant acts:

**Axiom 6** (Risk Independence - RI)**.** If $x, y, z \in X$ and $\gamma \in (0, 1]$, $x \succcurlyeq y$ implies $\gamma x + (1 - \gamma)z \succcurlyeq \gamma y + (1 - \gamma)z$.

The Archimedean axiom is then replaced by two standard axioms. The first is a continuity condition, which is slightly stronger than the previous axiom A. The second condition requires that every act have a certainty equivalent.

[16] Separability in the evaluation on bets on states or events follows precisely from the restriction of the Certainty Independence axiom of Gilboa and Schmeidler (1989) to binary acts. This is the Binary Certainty Independence axiom of Cerreia-Vioglio et al. (2011). In the Savage setting, this is the well-known P4 property.

**Axiom 7** (Continuity - C)**.** For any three acts $f, g, h \in \mathcal{F}$, the sets $\{\alpha \in [0,1] : \alpha f + (1-\alpha)g \succcurlyeq h\}$ and $\{\alpha \in [0,1] : h \succcurlyeq \alpha f + (1-\alpha)g\}$ are closed.

**Axiom 8** (Solvability - S)**.** For any $f \in \mathcal{F}$, there exists $x \in X$ such that $f \sim x$.

While very mild—and satisfied by the great majority of all the existing models of decision making—this axiom restricts the way in which a DM can evaluate bets with extremely high or extremely low payoffs, which is not the case for the SSEU model presented earlier. (See Remark 3 in Appendix A.11.) However, this feature is consistent with our objective of trying to encompass a wider range of attitudes towards the valence of an event.

We obtain the following result that generalizes Theorem 1. Here, we let $B_0(2^S, K)$ be the set of all simple $K$-valued functions, and say $I : B_0(2^S, K) \to \mathbb{R}$ is *normalized* if $I(\alpha) = \alpha$ for every $\alpha \in K$, and $I$ is *constant-Archimedean* if $\text{Range}(I) \subseteq K$.

**Theorem 3.** *The binary relation $\succcurlyeq$ satisfies axioms WO, RI, C, S and IS if and only if there exists a non-constant, affine function $u : X \to \mathbb{R}$ and a continuous, normalized, and constant-Archimedean functional $I : B_0(\Sigma, Range(u)) \to \mathbb{R}$ such that for every $f, g \in \mathcal{F}$, $f \succcurlyeq g$ if and only if $I(u(f)) \geq I(u(g))$.*

We say that a preference $\succcurlyeq$ has a *Bernoullian Continuous* (*BC*) *representation* if there are $u$ and $I$ satisfying the properties listed in Theorem 3. To illustrate the scope of the BC representation, consider any bet $xEy$ with $x \succ y$. The functional $I$ can be written as

$$I(u(xEy)) = \rho_{x,y}(E)u(x) + (1 - \rho_{x,y}(E))u(y),$$

where $\rho_{x,y}(E) = [I(u(xEy) - u(y)]/[u(x) - u(y)]$. It is easy to prove that, for each $x, y \in X$ with $x \succ y$, the function $\rho_{x,y} : 2^S \to \mathbb{R}$ is a *signed capacity*, that is, a set function satisfying $\rho_{x,y}(S) = 1$ and $\rho_{x,y}(\emptyset) = 0$.[17]

Paralleling our earlier decomposition of a signed probability measure $\nu$, a signed capacity $\rho_{x,y}$ can be decomposed into a capacity and a function reflecting (generalized) valence. That is, we can write

$$\rho_{x,y} = \mu_{x,y} + \Gamma_{x,y},$$

where $\mu_{x,y} : 2^S \to [0,1]$ is a capacity on $2^S$ and the function $\Gamma_{x,y} : 2^S \to \mathbb{R}$, which we call a *generalized valence*, satisfies (only) $\Gamma_{x,y}(\emptyset) = \Gamma_{x,y}(S) = 0$.

**Proposition 6.** *If the preference $\succcurlyeq$ has a BC representation, for each $x, y \in X$ with $x \succ y$, there is a capacity $\mu_{x,y}$ and a generalized valence $\Gamma_{x,y}$ such that*

$$I(u(xEy)) = \mu_{x,y}(E)u(x) + \Gamma_{x,y}(E)u(x) + (1 - \mu_{x,y}(E))u(y) + (1 - \Gamma_{x,y}(E))u(y).$$

[17] Indeed, if $E = S$, $xSy = x$ and so $I(u(xSy) = u(x)$, implying $\rho_{x,y}(S) = 1$. Similarly if $E = \emptyset$.

**Remark 2.** *It is straightforward to state a behavioral condition that restricts* $\Gamma_{x,y}$ *to obey the following natural property: For* $x \succ y$*, if* $\Gamma_{x,y}(A) \geq 0$ *then* $\Gamma_{x,y}(A^c) \leq 0$*. A sufficient condition is that if* $x \succ y$ *and* $xAy \succcurlyeq x$*, then* $y \succcurlyeq xA^c y$*.*

BC preferences can account for stake-dependent WTB. That is, assuming $X \subseteq \mathbb{R}$, we can accommodate preferences where, for some $x \succ y$,

$$y \succ xEy \quad \text{and} \quad (x+m)Ey \succ y$$

for some $m \in X$. When the stakes are small, the aversiveness of the event $E$ leads the DM to prefer the bet $xEy$ over obtaining $x$ for sure. However, for a sufficiently large payoff $x + m$, the DM reverses their preference. Indeed, assume that $\rho_{x,y}(E) + \Gamma_{x,y}(E) = \rho(E) + \gamma(E)/(|x| + |y|)$ for some capacity $\rho$ and valence function $\gamma$ (in the sense of Def. 2). If $\gamma(E) < 0$, then, for a sufficiently small $x$, we can have $u(y) > I(u(xEy))$. However, as $x$ increases to $x + m$, we get $\rho_{x+m,y}(E) + \Gamma_{x+m,y}(E) \approx \rho(E)$, so that $I(u((x + m)Ey)) > u(y)$.

Exploiting the non-additivity of the (signed) capacity $\rho_{x,y}$, the BC model can also reproduce the preference pattern observed experimentally by Schneider and Schonger (2019). They found evidence for violations of the following consequence of Monotonicity: If $xEz \succ yEz$ for some $x, y, z \in X$ and an event $E$, then $xEz' \succcurlyeq yEz'$ for all $z' \in X$.

**Example 6.** Consider the special case of the BC model in which the WTB $\rho_{x,y}$ is actually stake-independent; that is, $\rho_{x,y}(E) = \rho(E)$ for all $x, y \in X$ an all $E \subseteq S$. Fix payoffs $x, y \in X$ with $x \succ y$ and consider an event $E$ and a capacity $\rho$ with $\rho(E) > 0$. Given $z \in X$ with $y \succcurlyeq z$, we have

$$I(u(xEz)) = \rho(E)u(x) + (1 - \rho(E))u(z) > \rho(E)u(y) + (1 - \rho(E))u(z) = I(u(yEz)).$$

That is, $xEz \succ yEz$. Now suppose that $1 - \rho(E^c) < 0$. Then, given $z' \succcurlyeq x$, we have

$$I(u(yEz')) = \rho(E^c)u(z') + (1 - \rho(E^c))u(y) > \rho(E^c)u(z') + (1 - \rho(E^c))u(x) = I(u(xEz')).$$

That is, $yEz' \succ xEz'$. Notice that if the capacity $\rho$ is a signed measure, then the conditions $\rho(E) > 0$ and $1 - \rho(E^c) < 0$ cannot be simultaneously satisfied. Also, the capacity $\rho$ is not convex, since $\rho(E) + \rho(E^c) > 1$. △

# 7 Discussion

We conclude with some conceptual remarks and further comments on the literature.

**a. A Dutch Book Objection** There is a possible objection to our theory, which is that a DM following SSEU might be susceptible to a "Dutch Book," and a consequent money pump (de Finetti, 1937). If the DM's WTB on event $E$ is \$$\nu(E)$, where $\nu(E) > 1$, then a bettor who does not share this assessment will be able to obtain a sure win, by selling the DM a bet with prize

\$1 if $E$ obtains and \$0 otherwise—–at a price of $\$(\nu(E)-\epsilon)$ where $\epsilon < \nu(E)-1$. But, while this might be feasible as a one-time transaction, it is not clear to us that it will be repeatable. After the first transaction, the DM might well change valence so that $\nu(E) \leq 1$ for subsequent bets. Money pumps can occur only in the presence of unchanging preferences over the course of the pump, and we see no reason to assume that valence might not vary with the DM's betting history. Even for the one-time bet, the sure loss looks troublesome only from an outsider's perspective (assuming the outsider follows classical SEU). From the DM's perspective, the positive valence attached to $E$ implies a (non-monetary) welfare gain from betting on $E$.

**b. Related Models** There are other papers in decision theory where signed probabilities arise. Dekel et al. (2001, 2007) study preference over menus of lotteries and provide an axiomatization of an EU representation with respect to a signed probability measure on a subjective state space. In their framework, "positive" states represent potential future normative preferences, while "negative" states model possible future temptations that could affect preferences. This is a quite different interpretation from the one we offer here. Perea (2022) considers a setting motivated by game theory and axiomatizes a player's conditional preference relation over acts, where the conditioning is on different signed probability measures ("beliefs") the DM might hold over choices made by other players. Ke and Zhao (2023) present a model of decision making under ambiguity called "cautiously optimistic linear utility" which features a collection of sets of possibly signed (subjective) measures over states. Neither of these papers offer an interpretation for the signed probabilities they obtain.

Other models relax or omit altogether the Monotonicity assumption. Grant and Polak (2013) relax this axiom in order to represent general mean-dispersion aversion preferences, which include well-known non-monotonic models like mean-variance utility. In Ellis and Piccione (2017), Monotonicity fails because individuals can misperceive correlations between actions, for instance, by believing that mixing two perfectly correlated assets can reduce risk. De Waegenaere and Wakker (2001) and Bommier (2017) depart from Monotonicity in order to obtain non-separability of preferences over dates or states of the world, respectively. The SSEU model retains full separability.

**c. Valence and Complexity** Recent empirical and theoretical research suggests that aversion to complexity can explain violations of Monotonicity (e.g., de Clippel et al., 2024; Puri, 2024; Gu and Chan, 2024). In cases where the dominant act is perceived as "too complex" compared with a dominated one, complexity aversion may lead individuals to prefer the dominated option. However, it seems clear that complexity aversion cannot explain aversion to hedging, which involves binary—hence simple—lotteries. Another motivation for relaxing Monotonicity arises from failures in contingent or state-by-state thinking (e.g., Zhang and Levin, 2017; Echenique et al., 2022), which are compatible with the weaker notion of obvious dominance (Li, 2017).[18] Our

[18] Obvious dominance states that if, for all $s \in S$, $f(s) \succcurlyeq g(s')$ for all $s' \in S$, then $f \succcurlyeq g$, and $f \succ g$ if the first preference is strict (see Zhang and Levin, 2017).

SSEU preferences display state-by-state thinking, and may not respect obvious dominance. For example, if $x \succ y$ then $xEy$ obviously dominates $y$, but an SSEU DM may find $y \succ xEy$.

# A Appendix: Proofs

## A.1 Proof of Proposition 1

The first result provides an intermediate step in the proof of Proposition 1.

**Proposition 7.** *Assume that $\succcurlyeq$ admits the SSEU representation $(u, \nu)$. Then there exist $p^+, p^- \in \Delta^+(S)$, and $b \geq 0$ such that*

$$\nu(s) = (1+b)p^+(s) - bp^-(s),$$

*for every $s \in S$, so that for every $f \in \mathcal{F}$*

$$\int_S u(f)d\nu = (1+b)\int_S u(f)dp^+ - b\int_S u(f)dp^-.$$

*Moreover, $\succcurlyeq$ satisfies Axiom 4 if and only if $b = 0$.*

*Proof.* Let $a = \nu^+(S)$ and $b = \nu^-(S)$. Clearly $a \geq 0$, $b \geq 0$, and $\nu(S) = 1$ excludes that $a = b = 0$. If $b = 0$, then $\nu = \nu^+$ and $a = 1$. Moreover, if $a < 1$ then $1 = \nu(S) = a - b$ would imply $b \leq 0$, which contradicts the fact that $\nu^-$ is a positive measure. Thus, $a \geq 1$. By setting $p^+(s) = \frac{\nu^+(s)}{a}$ and $p^- = \frac{\nu^-(s)}{b}$ (if $b > 0$, otherwise $p^+ = \nu^+ = \nu$) for every $s \in S$, we have that $p^+, p^- \in \Delta^+(S)$. Moreover, we have

$$\nu(s) = \nu^+(s) - \nu^-(s) = \frac{a\nu^+(s)}{a} - \frac{b\nu^-(s)}{b} = ap^+(s) - bp^-(s)$$

for every $s \in S$. Now let

$$N' = \{s \in S : x\{s\}y \succ y \text{ for some } y \succ x\},$$

and observe that $s \in N'$ if and only if $\nu(s) < 0$. If $\succcurlyeq$ satisfies M, then $N' = \emptyset$, and since $b = \nu^-(S) = \nu^-(S \cap N) = \nu^-(\emptyset) = 0$. Hence $b = 0$ as desired. If $b = 0$, then $\succcurlyeq$ satisfies M. □

*Proof of Proposition 1.* By defining $\nu(s) = p(s) + \gamma(s)$, a valence representation can be rewritten as $V(f) = \int_S u(f)d(p+\gamma) = \int_S u(f)d\nu$. The set function $\nu$ is such that, for any $E, F \subseteq S$ with $E \cap F = \emptyset$, $\nu(E \cup F) = p(E \cup F) + \gamma(E \cup F) = p(E) + p(F) + \gamma(E) + \gamma(F) = \nu(E) + \nu(F)$, where the second equality follows from $p$ being a probability and Property 1 of $\gamma$ in Definition 2. Lastly, Condition 2 in Definition 2 implies that $\nu(S) = p(S) + \gamma(S) = 1 + 0 = 1$, so that $\nu \in \Delta(S)$.

For the opposite implication, suppose $\succcurlyeq$ has a SSEU representation $(u, \nu)$. By Proposition 7, there is $b \geq 0$, and $p^+, p^- \in \Delta^+(S)$ such that $\nu(s) = (1+b)p^+(s) - bp^-(s)$. The valence representation follows by defining $p(s) = p^+(s)$ and $\gamma(s) = b(p^+(s) - p^-(s))$. □

## A.2 Proof of Proposition 2

Observe that since by their definition $p^*(P) = 1$ and $\gamma^*(s) = bp^*(s)$ for all $s \in P$, we obtain that, for every $f, g, h \in \mathcal{F}$:

$$
\begin{aligned}
f \succsim_P g &\iff fPh \succsim gPh \\
&\iff \int_P u \circ f \, d\nu + \int_{P^c} u \circ h \, d\nu \geq \int_P u \circ g \, d\nu + \int_{P^c} u \circ h \, d\nu \\
&\iff \int_P u \circ f \, dp^* + \int_P u \circ f \, d\gamma^* \geq \int_P u \circ g \, dp^* + \int_P u \circ g \, d\gamma^* \\
&\iff (1+b) \int_P u \circ f \, dp^* \geq (1+b) \int_P u \circ g \, dp^* \\
&\iff \int u \circ f \, dp^* \geq \int u \circ g \, dp^*.
\end{aligned}
$$

This shows that $\succsim_P$ has a SEU representation $(u, p^*)$.

## A.3 Proof of Proposition 3

Given a signed measure $\nu$ with $\nu(S) = 1$, it is standard to prove that $\|\nu\|_v = \sum_{s \in S} |\nu(s)| = \sup_{E \subseteq S} [\nu(E) - \nu(E^c)]$. First, we define the set

$$
A_{\nu^+} = \left\{ p \in \Delta^+(S) : p \leq \nu^+ \right\}.
$$

Second, we show that $A_{\nu^+} = \operatorname{argmin}_{q \in \Delta^+(S)} \|\nu - q\|_v$. To prove this inequality, we show that $\|\nu - q\|_v \geq 2b$ for all $q \in \Delta^+(S)$. Indeed,

$$
\begin{aligned}
\|\nu - q\|_v =& \sup_{E \subseteq S} [\nu(E) - p(E) - \nu(E^c) + p(E^c)] \\
=& 2 \sup_{E \subseteq S} [\nu(E) - p(E)].
\end{aligned}
$$

Taking $E = P$, we get $\|\nu - q\|_v \geq 2(\nu(P) - q(P)) \geq 2(1 + b - 1) = 2b$. At the same time,

$$
\|\nu - q\|_v = \sum_{s \in P} |\nu(s) - q(s)| + \sum_{s \in N} |\nu(s) - q(s)|.
$$

Therefore,

$$
\begin{aligned}
\|\nu - q\|_v =& \sum_{s \in P} |\nu(s) - q(s)| + \sum_{s \in N} (\nu^-(s) + q(s)) \\
=& \sum_{s \in P} |\nu(s) - q(s)| + b + q(N) = \sum_{s \in P} |\nu(s) - q(s)| + 1 - q(P) \\
=& 1 + b + \sum_{s \in P} [|\nu(s) - q(s)| - q(s)].
\end{aligned}
$$

If $q \in A_{\nu^+}$, then $\|\nu - q\|_v = 1 + b + \sum_{s \in P} [\nu(s) - 2q(s)] = 1 + b + 1 + b - 2 = 2b$, thus $A_{\nu^+} \subseteq \operatorname{argmin}_{q \in \Delta^+(S)} \|\nu - q\|_v$. For the converse inclusion. Take $q \in \Delta^+(S)$ such that $\|\nu - q\|_v = 2b$.

Then, it must be that $q(P) = 1$, since otherwise $\|\nu - q\|_v = 2 + 2b - 2q(P) > 2b$, which is impossible. Therefore, $\sum_{s\in P}[|\nu(s) - q(s)| - q(s)] = 1 + b - 2 = \sum_{s\in P}[\nu(s) - 2q(s)]$, which implies

$$\sum_{s\in P}[|\nu(s) - q(s)|] = \sum_{s\in P}[\nu(s) - q(s)]$$

or

$$2\sum_{s\in P}(\nu(s) - q(s))^- = 0,$$

so that $q \in A_{\nu^+}$. Lastly, if $(p^*, \gamma^*)$ is the canonical decomposition of $\nu$, $p^* = \frac{\nu^+}{1+b} \leq \nu^+$, thus $p^* \in A_{\nu^+}$.

## A.4 Proof of Theorem 1

Several steps of this proof are standard, but we report them for completeness. We denote with $B$ the set of functions from $S$ to $\mathbb{R}$, and given $K \subseteq \mathbb{R}$ we denote with $B(K)$ the set of functions from $S$ to $K$.

Since $\mathcal{F}$ is mixture set and because $\succcurlyeq$ satisfies Axioms 1-3, by Theorem 8 in Herstein and Milnor (1953) there exists $V : \mathcal{F} \to \mathbb{R}$ that represents $\succcurlyeq$ such that

$$V(\alpha f + (1-\alpha)g) = \alpha V(f) + (1-\alpha)V(g),$$

for every $f, g \in \mathcal{F}$ and $\alpha \in [0,1]$. Moreover, $V$ is unique up to positive affine transformations. Further, observe that we can find $\bar{x}, \underline{x} \in X$ such that $V(\bar{x}) = -V(\underline{x}) = 1$. To see this, notice that there exist $\bar{x}, \underline{x} \in X$ such that $\bar{x} \succ \underline{x}$. Indeed, if $x \sim x'$ for every $x, x' \in X$, then it would follow by axiom 5 that $f \sim g$ for every $f, g \in \mathcal{F}$, contradicting Axiom 1. Therefore without loss of generality we can take $V$ such that $V(\bar{x}) = 1$ $V(\underline{x}) = -1$.

Define $u : X \to \mathbb{R}$ by $u(x) = V(x)$ for every $x \in X$. Let $K := u(X)$. Then $K$ is convex by the affinity of $V$ and without loss of generality satisfies $[-1, 1] \subseteq K$ by the previous paragraph.

Define a function $U : \mathcal{F} \to B(K)$ as follows: for every $f \in \mathcal{F}$ and $s \in S$

$$U(f)(s) := u(f(s)).$$

Then, $U$ is surjective since $K = u(X)$. Furthermore, the affinity of $V$ implies that for all $\alpha$ and $f, g \in \mathcal{F}$

$$U(\alpha f + (1-\alpha)g) = \alpha U(f) + (1-\alpha)U(g).$$

Now, define a functional $I$ on $B(K)$ by

$$I(\phi) = V\left(U^{-1}(\phi)\right),$$

for all $\phi \in B(K)$. Observe that $I$ is well defined since if $f, g \in \mathcal{F}$ satisfy $U(f) = U(g) = \phi$ then we have that $V(f) = V(g)$ by axiom 5. Clearly, it holds that for all $f \in \mathcal{F}$

$$I(U(f)) = I(u \circ f) = V(f).$$

Moreover, $I(x1_S) = x$ for every $x \in K$.

Now let $\phi, \psi \in B(K)$ and let $\alpha \in [0,1]$. Also, let $f, g \in \mathcal{F}$ be such that $U(f) = a$ and $U(g) = b$. Since $U$ is surjective, such $f$ and $g$ exist. Then, we obtain

$$\begin{aligned} I(\alpha a + (1-\alpha)b) &= V\left(U^{-1}(\alpha a + (1-\alpha)b)\right) \\ &= V\left(U^{-1}(\alpha U(f) + (1-\alpha)U(g))\right) \\ &= V\left(U^{-1}(U(\alpha f + (1-\alpha)g))\right) \\ &= V(\alpha f + (1-\alpha)g) \\ &= \alpha V(f) + (1-\alpha)V(g) \\ &= \alpha I(a) + (1-\alpha)I(b). \end{aligned}$$

It follows that $I$ is positively homogeneous, i.e. for every $\alpha > 0$ and $\phi \in B(K)$ such that $\alpha\phi \in B(K)$ it holds $I(\alpha\phi) = \alpha I(\phi)$. Moreover, $I$ is additive, that is for every $\phi, \psi \in B(K)$ such that $\phi + \psi \in B(K)$ it holds $I(\phi+\psi) = I(\phi) + I(\psi)$. To see this point, observe that by positive homogeneity we obtain $I(\phi + \psi) = I\left(2\left(\frac{\phi}{2} + \frac{\psi}{2}\right)\right) = 2I\left(\frac{\phi}{2} + \frac{\psi}{2}\right) = 2\left(\frac{I(\phi)}{2} + \frac{I(\psi)}{2}\right) = I(\phi) + I(\psi)$.

We can now extend $I$ to $B$ as follows. For every $\phi \in B$ there exists $\psi \in B(K)$ and $\alpha \geq 1$ such that $\psi = \alpha\phi$. We can therefore define $\bar{I} : B \to \mathbb{R}$ by

$$\bar{I}(\phi) = \alpha I(\psi).$$

Observe that it is now immediate to satisfies $\bar{I}$ satisfies for every $\alpha \in \mathbb{R}$ and $\phi, \psi \in B$

1. $\bar{I}(\phi + \psi) = \bar{I}(\phi) + \bar{I}(\psi)$;

2. $\bar{I}(\alpha\phi) = \alpha\bar{I}(\phi)$.

Now by the Riesz representation theorem (e.g., see Theorem 6.45 in Axler (1997)) there exists a (unique) $\nu : S \to \mathbb{R}$ such that for every $\phi \in B$

$$\bar{I}(\phi) = \int \phi d\nu = \sum_{s \in S} \phi(s)\nu(s).$$

We can therefore conclude that

$$f \succcurlyeq g \iff V(f) \geq V(g) \iff I(u(f)) \geq I(u(g)) \iff \int u(f)d\nu \geq \int u(g)d\nu.$$

Note that we can assume that $\nu(S) = \sum_{s\in S} \nu(s) = 1$. Indeed, it must be that $\sum_{s\in S} \nu(s) = \nu(S) \neq 0$ since $I(1_S) = 1$. Further, if $\sum_{s\in S} \nu(s) = \nu(S) \neq 1$, then letting let $\tilde{\nu}(s) = \frac{\nu(s)}{\nu(S)}$ and $\tilde{u} = \nu(S)u$, so that

$$V(f) = I(u(f)) = \int u(f)\nu = \int \tilde{u}(f)\tilde{\nu},$$

so that the claim is satisfied.

Finally, if there exists $\nu' \in \Delta(S)$ and an affine function $u' : X \to \mathbb{R}$ such that $(u', \nu')$ represents $\succsim$, then because $V$ is unique up to affine transformations there exists $a > 0$ and $b \in \mathbb{R}$ such that

$$\int u'(f)d\nu' = a \int u(f)d\nu + b$$

for every $f \in \mathcal{F}$ from which we obtain

$$u'(x) = au(x) + b,$$

for every $x \in X$. We can therefore conclude that $\nu' = \nu$ as desired.

## A.5 Proof of Corollary 1

$(i)$ implies $(ii)$. This implication follows directly by taking $\gamma = \gamma^*$. By Monotonicity, the SSEU representation is an SEU representation, and $\nu = p^+$, implying $b = 0$.

$(ii)$ implies $(iii)$. Since $\succsim$ has a valence representation $(u, p, \gamma)$ with $\gamma = 0$, for any $s \in S$ and $x \succ y$ we have $V(x\,\{s\}\,y) = p(s)(u(x) - u(y)) + u(y) \geq u(y)$ so that $x\,\{s\}\,y \succsim y$. Thus, $S \subseteq P$ and since $P \subseteq S$, we obtain $P = S$.

$(iii)$ implies $(i)$. Consider the binary relation $\succsim_P$ defined as $f \succsim_P g$ if and only if $fPh \succsim fPh$ for some $h \in \mathcal{F}$. By Proposition 2, $\succsim_P$ has a SEU representation $(u, p^*)$. If $P = S$, then $f \succsim_S g$ is equivalent to $fSh \succsim gSh$ and to $f \succsim g$. By Proposition 2, $\succsim$ has a SEU representation so it satisfies axiom M.

## A.6 Proof of Proposition 4

Suppose that $x \succ y$ and $y \succ x\{s\}y$. Then, $u(y) > p^*(s)(u(x) - u(y)) + u(y) + \gamma^*(s)(u(x) - u(y),$ which implies $0 > p^*(s)\,(u(x) - u(y)) + \gamma^*(s)\,(u(x) - u(y))$. Since $u(x) - u(y) > 0$, the previous inequality implies $\gamma^*(s) < 0$. As for the converse, since $\gamma^*(s)$ is either $bp^*(s) > 0$ or $-bp^-(s) < 0$, if $\gamma^*(s) < 0$, then $\gamma^*(s) = -bp^-(s)$ and $p^*(s) = 0$. Therefore $p^*(s)(u(x) - u(y)) = 0$, and using the fact that $u(x) > u(y)$, we obtain that $p^*(s)\,(u(x) - u(y)) + u(y) + \gamma^*(s)\,(u(x) - u(y)) = u(y) + \gamma^*(s)(u(x) - u(y)) < u(y)$. We can therefore conclude that $y \succ x\,\{s\}\,y$.

Now suppose that $x \succ y$ and $x\{s\}y \succ y$. Then, $p^*(s)(u(x)-u(y))+u(y)+\gamma^*(s)\,(u(x) - u(y)) > u(y)$, which implies $p^*(s)(u(x) - u(y)) + \gamma^*(s)(u(x) - u(y)) > 0$. Since $u(x) - u(y) > 0$, the previous inequality implies $p^*(s) + \gamma^*(s) > 0$. Since $\gamma^*(s)$ is either $bp^*(s) > 0$ or $-bp^-(s) < 0$, if $p^*(s) > 0$, then $\gamma^*(s) = (1 + b)p^*(s) > 0$. If $p^*(s) = 0$, then $\gamma^*(s) = -bp^-(s) < 0$, contradicting $p^*(s) + \gamma^*(s) > 0$. As for the converse, since $\gamma^*(s)$ is either $bp^*(s) > 0$ or $-bp^-(s) < 0$, if $\gamma^*(s) > 0$, then $\gamma^*(s) = bp^*(s)$. Therefore $p^*(s)(u(x) - u(y)) + \gamma^*(s)(u(x) - u(y)) > 0$, or equivalently, $p^*(s)(u(x) - u(y)) + u(y) + \gamma^*(s)(u(x) - u(y)) > u(y)$. Hence we obtain $x\,\{s\}\,y \succ y$ as desired.

## A.7 Proof of Proposition 5

It is easy to see that $E \subseteq S$ is $\succcurlyeq$-null iff $\nu(E) = p^*(E) + \gamma^*(E) = 0$. This immediately implies the characterization of $\succcurlyeq$-completely null events, by additivity of $\nu$.

Next, using the definitions of $p^+$ and $p^-$ from Proposition 7, we see that for every $s \in S$ we have $\nu(s) = 0$ iff $p^*(s) = p^+(s) = 0$ and $p^-(s) = 0$. (For the "only if" direction, notice that by definition $\nu(s) = 0$ implies $\nu^+(s) = 0 = \nu^-(s)$, which immediately implies $p^+(s) = 0$ and $p^-(s) = 0$.) Analogously, $\nu(s) > 0$ iff $p^*(s) = p^+(s) > 0$ and $p^-(s) = 0$, and $\nu(s) < 0$ iff $p^*(s) = p^+(s) = 0$ and $p^-(s) > 0$. If $E$ is $\succcurlyeq$-classically null, it cannot contain any state $s$ such that $x\{s\}y \succ y$ for some $x \succ y$ (which would imply $\nu(s) > 0$), hence $p^*(E) = 0$. Conversely, if $p^*(E) = 0$, $E$ cannot clearly contain any event $F$ such that $p^*(F) > 0$, and so it is $\succcurlyeq$-classically null.

## A.8 Proof of Theorem 2

$(i) \implies (ii)$: First observe that 1 and 2 admit SSEU representations given by $(u, \nu_1), (u, \nu_2)$ such that $u_1 = u_2$. To see this, observe that since 1 is more classical than 2, taking $f = x, g = y$ it follows that

$$x \succcurlyeq_1 y \iff xEh \succcurlyeq_1 yEh \iff xEh \succcurlyeq_2 yEh \iff x \succcurlyeq_2 y,$$

for all $x, y \in X$. Further observe that $P_1 = P_2 := P$. To see this, take $s \in P_1$. Then it must be that $x\{s\}y \succcurlyeq_1 y$. Since 1 is more classical than 2, it follows that letting $x\{s\}y \succcurlyeq_2 y$, which implies $s \in P_2$, so that $P_1 \subseteq P_2$. One can show in the same way that $P_2 \subseteq P_1$. It follows that $N_1 = N_2 := N$.

Now define $\succcurlyeq'_i$, $i = 1, 2$ on $X^P$ as follows

$$f \succcurlyeq'_i g \iff \text{there exists } h \in \mathcal{F} \text{ such that } fPh \succcurlyeq_i gPh,$$

for every $f, g \in X^P$. Since 1 is more classical than 2, we obtain that

$$f \succcurlyeq'_1 g \iff f \succcurlyeq'_2 g,$$

it follows that

$$p_1^*(s) = \frac{\nu_1(s)}{\nu_1(P)} = \frac{\nu_2(s)}{\nu_2(P)} = p_2^*(s) = p^*(s) \text{ for every } s \in P.$$

Finally, given any $s \in N$, choose $x, y, z$ such that $x\{s\}y \sim_1 z$.[19] Since 1 is more classical than 2 it follows that

$$\nu_1(s) = \frac{u(y) - u(z)}{u(x)} \leq \nu_2(s),$$

which implies that $\gamma_1^*(s) \leq \gamma_2^*(s)$ whenever $s \in N$. When $s \in P$, $\gamma_1^*(s) = b_1 p^*(s) \geq b_2 p^*(s) = \gamma_2^*(s)$. We can therefore conclude that $|\gamma_1^*| \geq |\gamma_2^*|$.

[19]Such triple $x, y, z$ always exist. Let $y$ be such that $u(y) = 0$ and choose $x, z$ so that $\nu_1(s) = \frac{u(z)}{u(x)}$.

$(ii) \implies (i)$: Assume that $p_1^* = p_2^* = p^*$. Take $f, g, h \in \mathcal{F}$ and $E \subseteq P_1 \cup P_2$. We have

$$fEh \succsim_1 gEh \iff \int_S u(f)dp^* \geq \int_S u(g)dp^* \iff fEh \succsim_2 gEh.$$

Now take $x, y, z$ with $u(x) > u(y)$ and $E \subseteq N$ such that

$$xEy \sim_1 z,$$

which is equivalent to $\gamma_1^*(E) = \frac{u(z)}{u(x)-u(y)}$. Since $E \subseteq N$, it follows that $\gamma_2^*(E) \geq \gamma_1^*(E)$, which is equivalent to

$$xEy \succsim_2 z,$$

as desired.

## A.9 Proof of Fact 3

According to SSEU, the difference between $V(f_2)$ and $V(f_1)$ is

$$u(x-\psi) - \nu(L)u(x-\ell) - (1-\nu(L))u(x).$$

Since $0 \leq \nu(L) \leq 1$, Jensen's inequality yields

$$V(f_2) - V(f_1) = u(x-\psi) - \nu(L)u(x-\ell) - (1-\nu(L))u(x) \geq$$
$$u(x-\psi) - u(x-\nu(L)\ell) = 0.$$

Thus, the DM buys the insurance policy. The SSEU difference between $f_4$ and $f_2$ is

$$V(f_4) - V(f_2) = \nu(E)u(x-\psi-\pi+w) + (1-\nu(E))u(x-\psi-\pi) - u(x-\psi).$$

By assumption, $\nu(E) = p(E) + \gamma(E) = 1 + \epsilon$ for some $\epsilon \geq 0$. Also, $u(x-\psi-\pi+w) - u(x-\psi) \geq 0$ since $u$ is increasing and $w \geq \pi$. Similarly, $u(x-\psi-\pi+w) - u(x-\psi-\pi) \geq 0$. Therefore

$$u(x-\psi-\pi+w) + \epsilon(u(x-\psi-\pi+w) - u(x-\psi-\pi)) - u(x-\psi) \geq 0,$$

because $w \geq \pi$, and thus $V(f_4) \succsim V(f_2)$. We conclude that $V(f_4) \succsim V(f_2) \succsim V(f_1)$.

## A.10 Proof of Fact 4

If $p(E) + \gamma(E) \geq 1$, then $\gamma(E) \geq 1 - p(E)$. Since $u(x) \geq u(y)$, we have $\gamma(E)[u(x) - u(y)] \geq (1-p(E))[u(x) - u(y)]$. Adding and subtracting $p(E)u(x)$ on the right-hand side implies

$$\gamma(E)[u(x) - u(y)] \geq u(x) - [p(E)u(x) + (1-p(E))u(y)].$$

Since $u$ is increasing, we can write $u(x) \geq u(p(E)x + (1-p(E))y)$, and thus

$$\gamma(E)[u(x) - u(y)] \geq u(p(E)x + (1-p(E))y) - [p(E)u(x) + (1-p(E))u(y)],$$

as required.

## A.11 Proof of Theorem 3

Necessity of the axioms is straightforward. As to sufficiency, since $X$ is mixture set and since $\succsim$ satisfies axioms WO, C and RI, by Theorem 8 in Herstein and Milnor (1953) there exists $u : X \to \mathbb{R}$ that represents the restriction of $\succsim$ on $X$ and such that

$$u\left(\alpha x + (1-\alpha)y\right) = \alpha u(x) + (1-\alpha)u(y),$$

for every $x, y \in X$ and $\alpha \in [0,1]$. Moreover, $u$ is unique up to positive affine transformations. Axiom S implies that each act $f$ has a certainty equivalent $x_f \in X$.

Now, define a functional $I : B_0(2^S, \text{Range}(u)) \to \mathbb{R}$ as $I(a) = u(x_f)$ where $u(f)(s) = a(s)$. To see that $I$ is well-defined, suppose that $u(f(s)) = a(s) = b(s) = u(g(s))$ for all $s \in S$. Then by IS $f \sim g$, therefore $I(a) = u(x_f) = u(x_g) = I(b)$, so $I$ is well-defined. By construction $I$ is normalized and constant-archimedean, and it can be proved that it is also continuous (w.r.t. to the sup norm).

**Remark 3.** *As mentioned in the main text, Solvability may impose restrictions on the way in which our DM can assign valence to events when assessing bets with very high or very low payoffs. This will be the case if the utility $u$ has a bounded range. The only (non-SEU) case in which Solvability imposes no restrictions is when $\text{Range}(u) = \mathbb{R}$, which describes exactly the intersection of the BC model and the SSEU model.*[20]

## A.12 Proof of Proposition 6

Suppose that $\succsim$ has a BC representation. Let define, for $x, y \in X$ with $x \succ y$, the set function $\rho_{x,y} : 2^S \to \mathbb{R}$ as $\rho_{x,y}(E) = \frac{I(u(xEy)) - u(y)}{u(x) - u(y)}$. By definition, $\rho_{x,y}(S) = 1$ and $\rho_{x,y}(\emptyset) = 1 - \rho_{x,y}(S) = 0$, thus $\rho$ is a signed capacity. By Cerreia-Vioglio et al. (2012, Proposition 7), any signed capacity of bounded variation $\rho_{x,y}$ can be decomposed into $\rho_{x,y} = \rho^+_{x,y} - \rho^-_{x,y}$, where $\rho^+_{x,y}$ and $\rho^-_{x,y}$ are capacities. Our signed capacity $\rho_{x,y}$ is clearly of bounded variation since the state space is finite. Thus, we can write $\rho_{x,y} = a_{x,y}\mu^+_{x,y} - b_{x,y}\mu^-_{x,y}$, where $a_{x,y} = \rho^+_{x,y}(S)$, $\mu^+_{x,y} = \rho^+_{x,y}/a_{x,y}$, $b_{x,y} = \rho^-_{x,y}(S)$, and $\mu^-_{x,y} = \rho^-_{x,y}/b_{x,y}$. With these definitions, we get $\mu^+_{x,y}(S) = \mu^-_{x,y}(S) = 1$, and the normalization $\rho_{x,y}(S) = 1$ implies $a_{x,y} - b_{x,y} = 1$. To obtain the decomposition above, we then set $\Gamma_{x,y}(E) = b_{x,y}(\mu^+_{x,y}(E) - \mu^-_{x,y}(E))$ for any $E \subseteq S$. Thus, for all $x \succ y$ in $X$ and all $E \subseteq S$, $I(u(xEy)) = \rho_{x,y}(E)u(x) + (1 - \rho_{x,y}(E))u(y)$ or, equivalently, $I(u(xEy)) = (\mu^+_{x,y}(E) + \Gamma_{x,y}(E))u(x) + (1 - \mu^+_{x,y}(E) - \Gamma_{x,y}(E))u(y)$ yielding the generalized valence representation.

[20]Given an SSEU preference $(u, \nu)$ with $\text{Range}(u)$ bounded, suppose that $E \subseteq S$ is such that $\nu(E) = 1 + \alpha$ for some $\alpha > 0$ and choose a normalization of $u$ such that $u(x_0) = 0$ for some $x_0 \in X$. Then if $\sup(\text{Range}(u)) = M$, there is going to be a $u(z) = M - \epsilon$ such that the bet $zEx_0$ has $V(zEx_0) = (M - \epsilon)(1 + \alpha) > M$, so that it does not have a certainty equivalent.

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