Epistemic Game Theory

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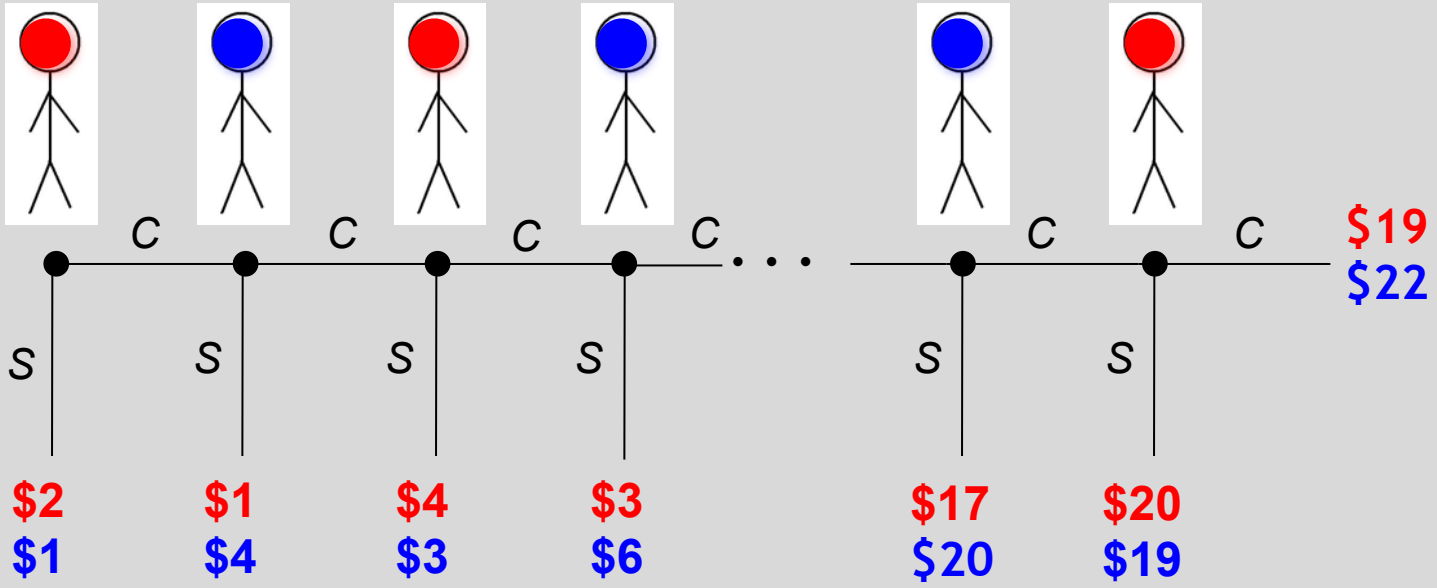
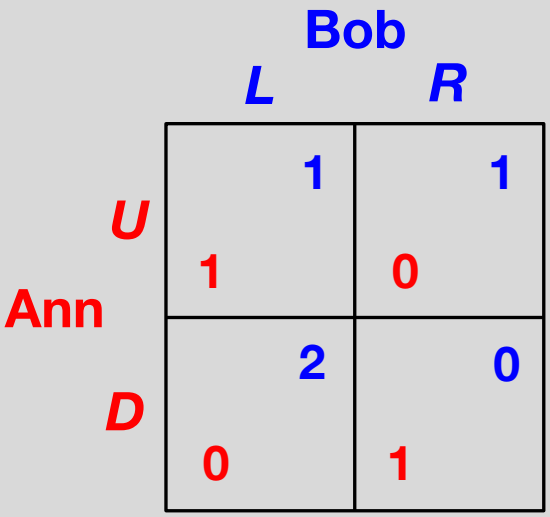
Some Paradoxes in Game Theory



"Consider player 1's initial decision to say stop. For this to be rational, player 1 must be pretty sure that if instead she says continue, player 2 will say stop at her first turn …. [But] might player 2 respond to player 1 saying continue by also saying continue? … [Because] once she sees that player 1 has chosen continue — an event that should never happen … — she might entertain the possibility that player 1 is not rational …. If, as a result, she thinks that player 1 would say continue at her next move if given the chance, then player 2 would want to say continue herself." *

Rosenthal, R., "Games of Perfect Information, Predatory Pricing, and the Chain Store," *Journal of Economic Theory*, 1980, 25, 92-100;
* Mas-Colell A., M. Whinston, and J. Green, *Microeconomic Theory*, Oxford University Press, 1995

Paradoxes contd.



"[T]he argument for deletion of a weakly dominated strategy for player *i* is that he contemplates the possibility that every strategy combination of his rivals occurs with positive probability. However, this hypothesis clashes with the logic of iterated deletion, which assumes, precisely, that eliminated strategies are not expected to occur." *

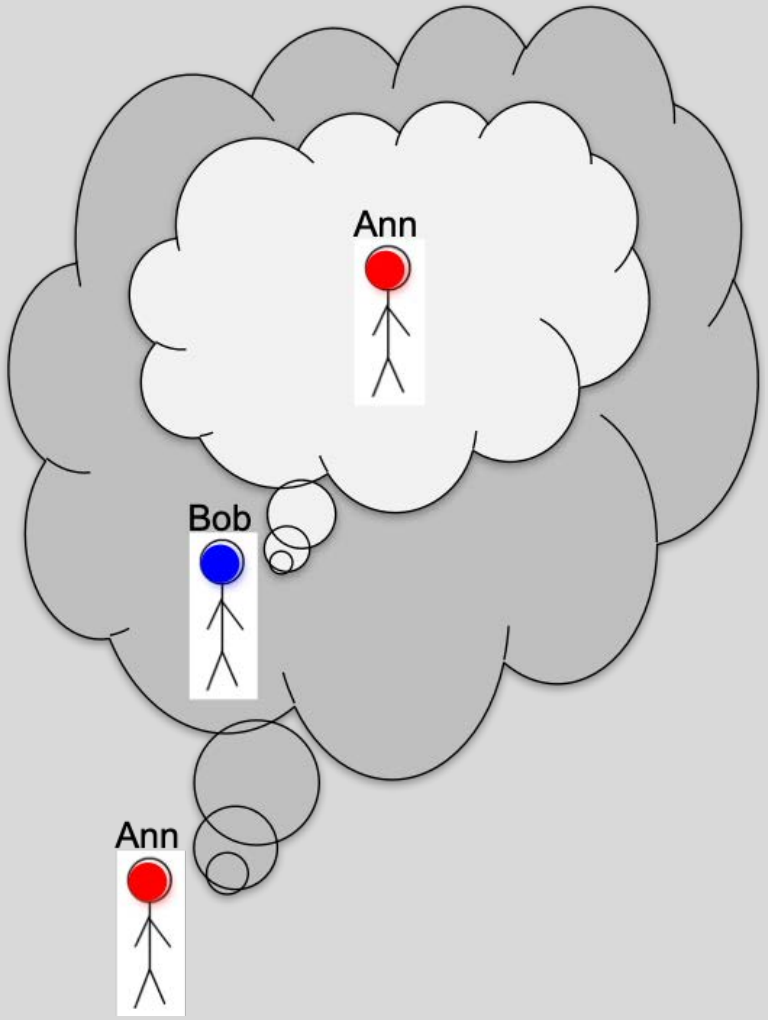
* Mas-Colell A., M. Whinston, and J. Green, *Microeconomic Theory*, Oxford University Press, 1995

Paradoxes contd.

Ann thinks Bob thinks
Ann thinks what Bob thinks
(about the game) is wrong

Does Ann think
what Bob thinks is wrong?

Does Ann not think
what Bob thinks is wrong?



* Brandenburger, A., and H.J. Keisler, "An Impossibility Theorem on Beliefs in Games," *Studia Logica*, 84, 2006, 211-240

What the Paradoxes Reveal

"Whenever, in any discipline, we discover a problem that cannot be solved within the conceptual framework that supposedly should apply, we experience shock. The shock may compel us to discard the old framework and adopt a new one." *****

Conventional game theory is inadequate to understand

the exact meaning of rationality in a game

the implications of rationality and belief in rationality

the construction of hierarchies of beliefs for the players

…

Epistemic game theory was built to address these problems

* Rapaport, A., "Escape from Paradox," *Scientific American*, 217, 1967, 50-56

An Epistemic Game

	L	R
U	2, 2	0, 0
D	0, 0	1, 1

$\lambda^a(t^a)$

T^b \ S^b	L	R
u^b	0	1/2
t^b	0	1/2

$\lambda^a(u^a)$

T^b \ S^b	L	R
u^b	1/2	0
t^b	0	1/2

$\lambda^b(t^b)$

T^a \ S^a	U	D
u^a	0	1/2
t^a	0	1/2

$\lambda^b(u^b)$

T^a \ S^a	U	D
u^a	1/2	0
t^a	0	1/2

Epistemic type spaces T^a, T^b

with associated maps $\lambda^a : T^a \to \mathscr{M}(S^b \times T^b)$

$\lambda^b : T^b \to \mathscr{M}(S^a \times T^a)$

An Epistemic Game contd.

	L	R
U	2, 2	0, 0
D	0, 0	1, 1

$\lambda^a(t^a)$

T^b	L	R
u^b	0	1/2
t^b	0	1/2

S^b

$\lambda^a(u^a)$

T^b	L	R
u^b	1/2	0
t^b	0	1/2

S^b

$\lambda^b(t^b)$

T^a	U	D
u^a	0	1/2
t^a	0	1/2

S^a

$\lambda^b(u^b)$

T^a	U	D
u^a	1/2	0
t^a	0	1/2

S^a

At the state (D, t^a, R, t^b)

Ann is 'correct' about Bob's strategy

Bob is correct about Ann's strategy

Ann, though, thinks it possible Bob is wrong about her strategy

Ann is rational

Bob is rational

Ann, though, thinks it possible Bob is irrational

Features of Epistemic Game Theory

We can see from the example that

- an epistemic type structure is a descriptive not a predictive tool
- epistemic game theory allows for ‘incorrect’ as well as ‘correct’ beliefs
- epistemic game theory allows for both rationality and irrationality

These are typical features of the epistemic approach

What Epistemic Game Theory Can Do

Resolve paradoxes by giving

well-defined epistemic conditions yielding backward induction (Battigalli and Siniscalchi, 2002)

well-defined epistemic conditions yielding iterated (weak) dominance (Brandenburger, Friedenberg, Keisler, 2008)

well-defined models of ‘all' possible beliefs, beliefs about beliefs, etc. (Mertens and Zamir, 1985; Brandenburger and Dekel, 1993)

But epistemic game theory also uncovers new phenomena!

Battigalli, P., and M. Siniscalchi, “Strong Belief and Forward-Induction Reasoning,” *Journal of Economic Theory*, 106, 2002, 356-391; Brandenburger, A., A. Friedenberg, and H.J. Keisler, “Admissibility in Games,” *Econometrica*, 76, 2008, 307-352; Mertens, J.-F., and S. Zamir, “Formulation of Bayesian Analysis for Games with Incomplete Information,” *International Journal of Game Theory*, 14, 1985,1-29; Brandenburger, A., and E. Dekel, “Hierarchies of Beliefs and Common Knowledge,” *Journal of Economic Theory*, 59, 1993, 189-198

New Phenomenon: Correlation

	L	*R*
U	1, 1, 3	1, 0, 3
D	0, 1, 0	0, 0, 0

X

	L	*R*
U	1, 1, 2	0, 0, 0
D	0, 0, 0	1, 1, 2

Y

	L	*R*
U	1, 1, 0	1, 0, 0
D	0, 1, 3	0, 0, 3

Z

The strategy *Y* is optimal for Charlie if she puts probability ½ : ½ on (*U*, *L*) : (*D*, *R*)

So this strategy is undominated (actually, iteratively undominated)

There are no independent probabilities under which *Y* is optimal

Still, *Y* can be justified epistemically if Charlie believes Ann's and Bob's hierarchies of beliefs are correlated

But this justification is not possible for all iteratively undominated strategies

New Phenomenon: Correlation contd.

The implication is that

rationality, belief in rationality, etc. in games with three or more players is not characterized by iterated dominance

because

iterated dominance contains ‘too much’ correlation

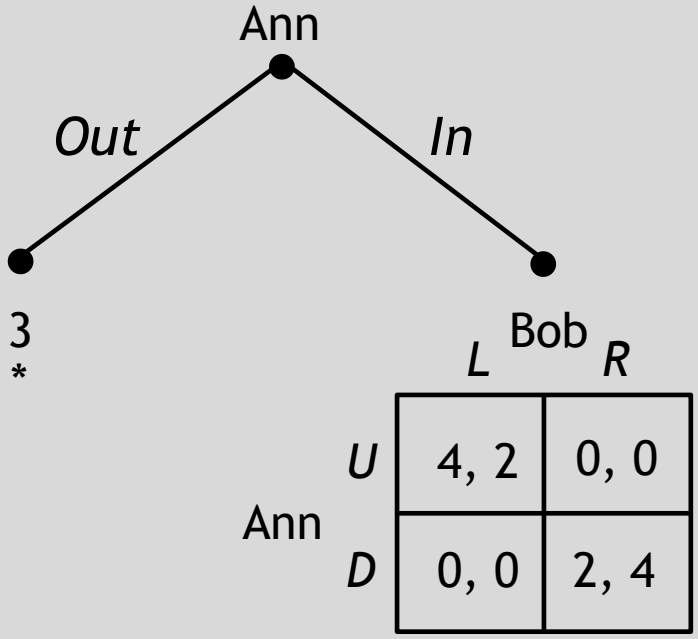
The implication of assuming

rationality, belief in rationality, etc. in games with three or more players

is open

indicating how young is the field of epistemic game theory

New Phenomenon: Context



Iterated weak dominance implies Ann plays *In-U* and Bob plays *L* ("forward induction")

But suppose Bob is expected to play *R* (he is a 'bully')

Then rationality, belief in rationality, etc. is consistent with Ann's playing *Out*

Kohlberg E., and J.-F. Mertens, "On the Strategic Stability of Equilibria," *Econometrica*, 54, 1986, 1003-1037;
Battigalli, P., and A. Friedenberg, "Forward Induction Reasoning Revisited," *Theoretical Economics*, 7, 2012, 57-98

New Phenomenon: Context contd.

The implication is that

> rationality, belief in rationality, etc. in game trees is not characterized by iterated weak dominance

because

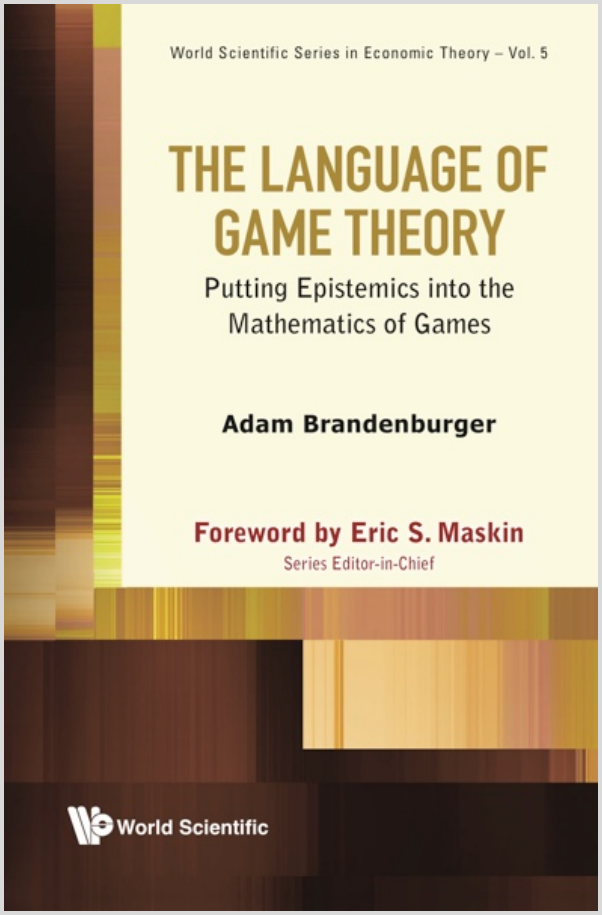
> iterated weak dominance does not capture context

The implication of assuming

> rationality, belief in rationality, etc. in game trees

is a new solution concept

> called an "*m*-best response sequence" (Brandenburger, Danieli, and Friedenberg, 2019)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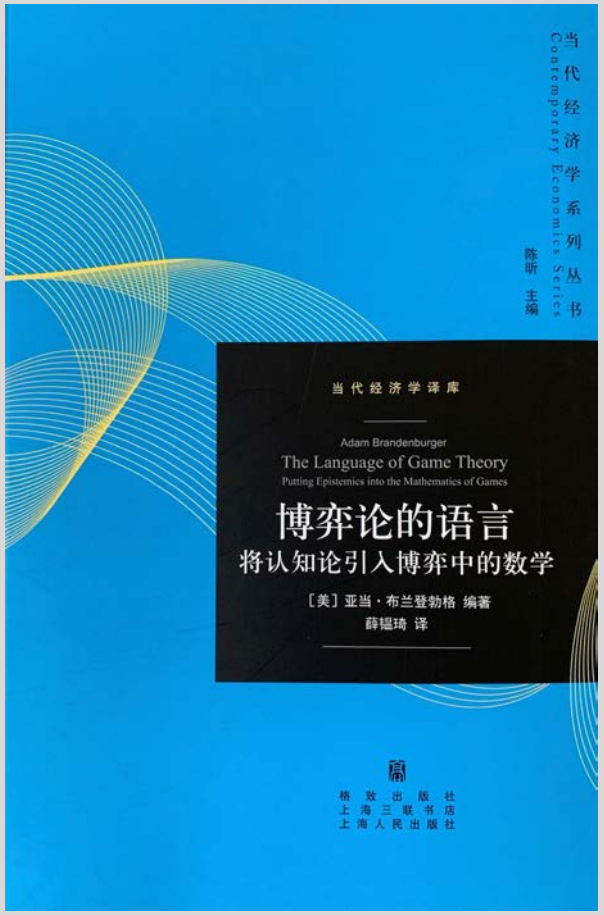
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Summary

Perhaps the biggest difference between epistemic game theory and conventional game theory is that the players' beliefs now become an input into the analysis, not an (equilibrium) output

This changes the very definition of a game to include not only the strategy sets and payoff functions (and tree structure), but also the players' hierarchies of beliefs

This aligns with the 'trilogy' of decision theory — choice set, utility function, and probability measure

Epistemic game theory was built to solve problems in conventional game theory, but it has become an approach in its own right

How will epistemic game theory connect to the cognitive sciences?