Introduction to Game Theory:

Two-by-Two Games

Version 10/29/17

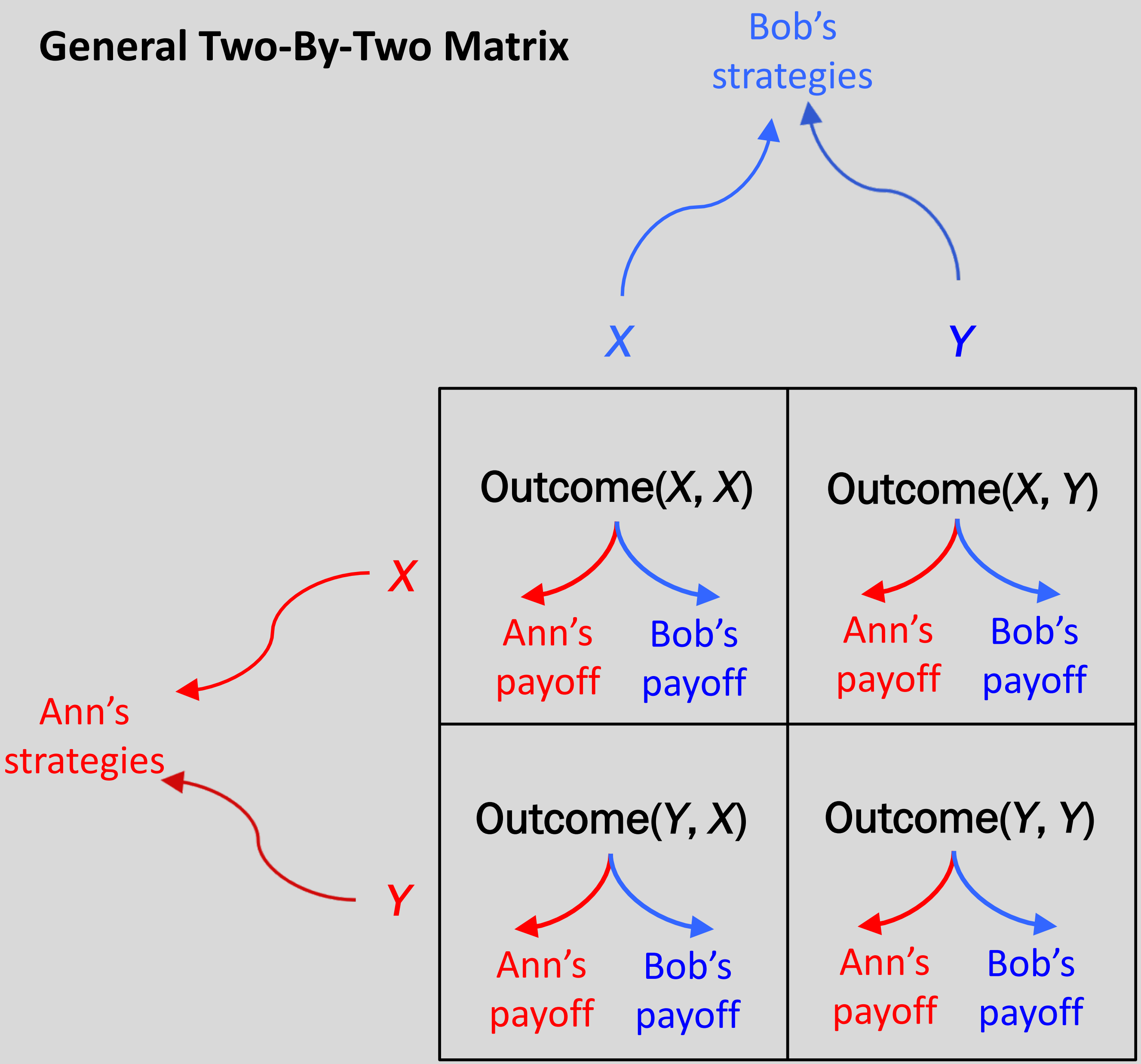
Sticklebacks

	Inspect predator	Don't inspect predator
Inspect predator	$B - C/2$ (blue); $B - C/2$ (red)	B (blue); $B - C$ (red)
Don't inspect predator	$B - C$ (blue); B (red)	0 (blue); 0 (red)

If $C > B > C/2 > 0$, what game is this?

Milinksi, M., "TIT FOR TAT in Sticklebacks and the Evolution of Cooperation," *Nature*, 325, 1987, 433-435; image from http://dbatm.mbc.nctu.edu.tw/download/species_figure/Gasterosteus_aculeatus.jpg

General Two-By-Two Matrix

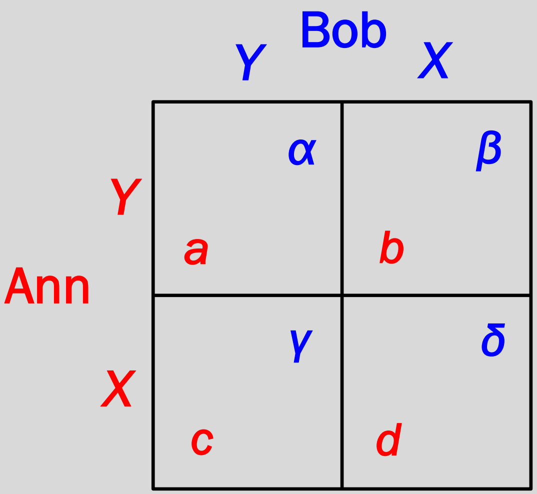


1. Payoffs are player-specific evaluations of outcomes

2. What if payoffs are not 'transparent' to the players? Later …

The Number of Two-By-Two Matrices

How many 2 × 2 matrix games are there?



Let's simplify the question:

Assume there are no ties among *a*, *b*, *c*, *d*, and no ties among α, β, γ, δ

So, Ann (resp. Bob) has a strict ranking of *a*, *b*, *c*, *d* (resp. α, β, γ, δ)

How many rankings of *a*, *b*, *c*, *d* (resp. α, β, γ, δ) are there?

The Number of Two-By-Two Matrices Cont'd

If we distinguish games via the players' ordinal rankings of payoffs, how many different 2 × 2 matrix games are there?

We can consider as strategically equivalent any two matrices where one can be obtained from the other by: (1) interchanging rows, (2) interchanging columns, (3) interchanging players, (4) any sequence of these operations

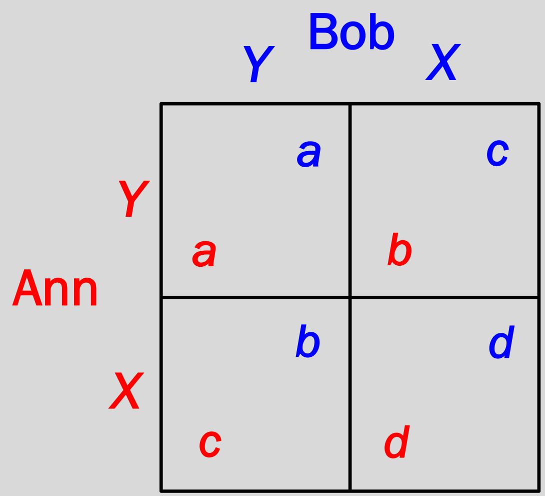
After this reduction, we would obtain 78 strategically distinct 2 × 2 matrix games*

This is still too many to remember!

Let's take a more heuristic approach …

* Rapoport, A., and M. Guyer, "A Taxonomy of 2 x 2 Games," *General Systems*, 11, 1966, 203-214

Symmetric Two-By-Two Matrices

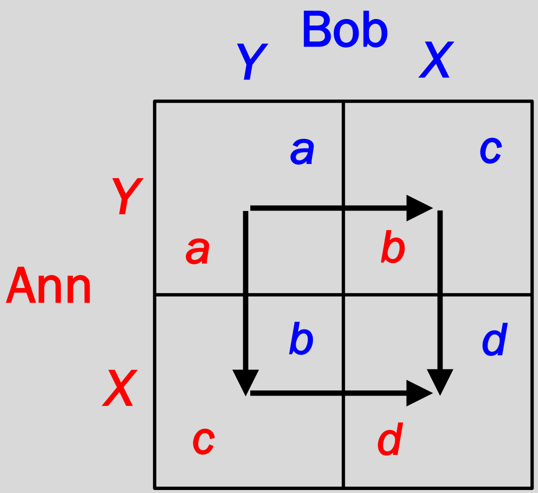


We will rank only *a* with respect to *c*, and *b* with respect to *d*

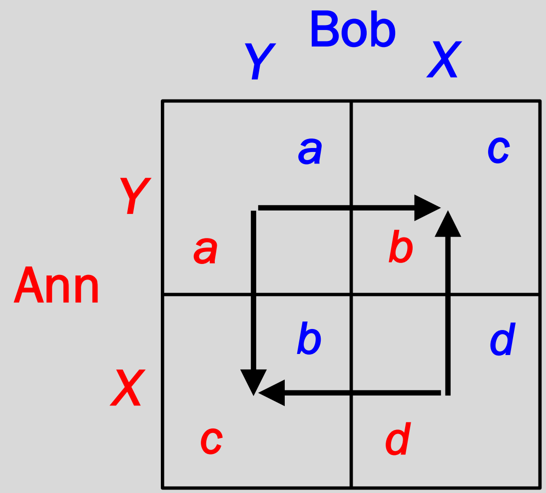
In this scheme, how many distinct symmetric 2 × 2 matrix games are there?

The approach is due to Ken Corts

A Scheme of Four Symmetric Two-By-Two Matrices

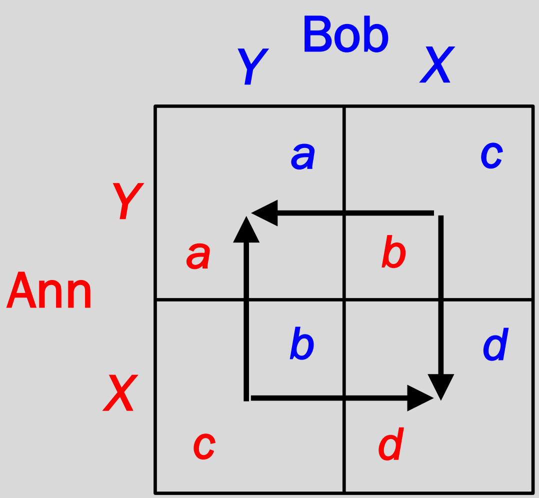


Prisoner's Dilemma

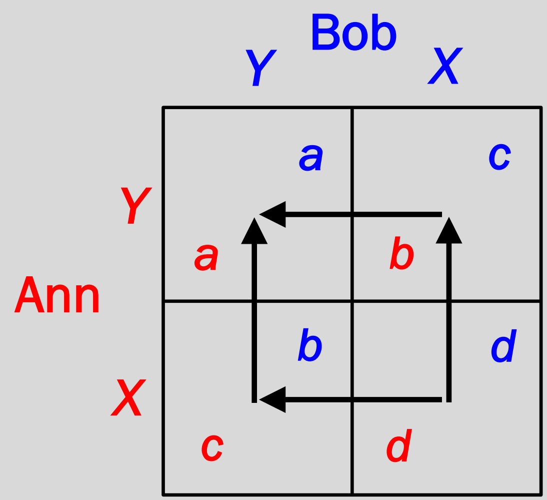


Battle of the Sexes

We assume that $a > d$ (this is just a choice of orientation)

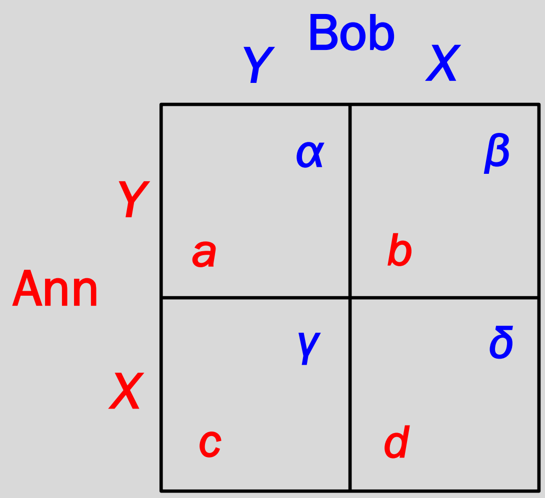


Coordination Game



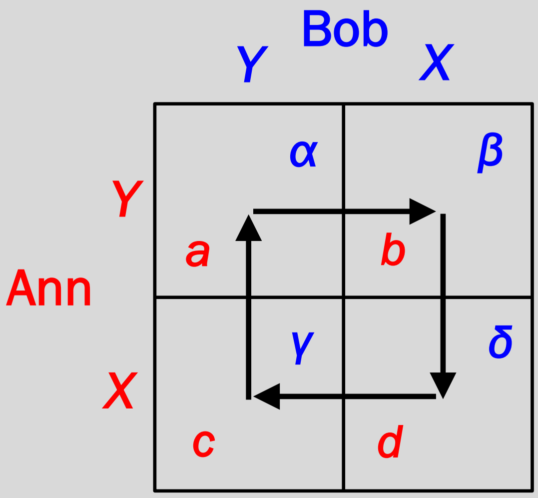
Prisoner's Delight

Asymmetric Two-By-Two Matrices

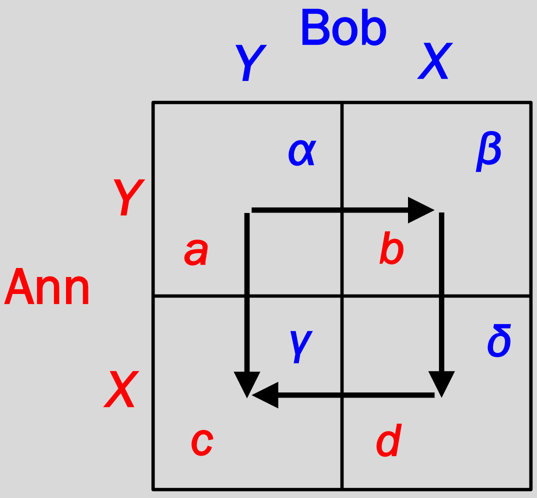


What conceptually new behavior of the arrows arises in the asymmetric case?

A Scheme of Two Asymmetric Two-By-Two Matrices



Matching Pennies



One-Sided Game

Preview of Analysis of Game Matrices

In the Prisoner's Dilemma, the strategy *X* for Ann is **dominant** (also, undominated) and the strategy *Y* is **dominated**; and likewise for Bob

In the Battle of the Sexes and the Coordination Game, there are no dominance relationships

In the Battle of the Sexes, the pairs of strategies (*X*, *Y*) and (*Y*, *X*) constitute **Nash equilibria**

In the Coordination Game, the pairs of strategies (*X*, *X*) and (*Y*, *Y*) constitute **Nash equilibria**

In Matching Pennies, there is no Nash equilibrium (in "pure" strategies)

In the One-Sided Game, the strategy *X* for Ann is dominant and the strategy *Y* for Bob is **iteratively undominated**