Symmetry and the Sixth Force: The Essential Role of Complements Online Appendix*

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This section presents a formal model to support our Example 1 in Section III of the main text. Customers are uniformly located on the line segment $[0, 1]$. There are two incumbent firms. Firm 0 is located at 0 and firm 1 is located at 1. Initially, there are only branded products available in the market. The customer located at x has a willingness-to-pay $v(x) - x$ for the branded product from firm 0 and a willingness-to-pay $v(x) - |1 - x|$ from firm 1. We assume that, for all customers $x \in [0, 0.3)$ and $x \in (0.7, 1]$, the gross willingness-to-pay is 3. For all customers $x \in [0.3, 0.7]$, the gross willingness-to-pay is 2.

When an unbranded substitute arrives in the market, two firms, both located at 0.5, offer identical versions of this product. The customer located at x has a willingness-to-pay $w(x) - |x - 0.5|$, where $w = 0.92$ for all customers.

The idea of the model is that when the unbranded substitute enters the market, it captures all the price-sensitive customers in the "middle" of the market. The remaining customers at the "ends" of the market have a strong preference for one or other branded product. We will see that the outcome is that the (equal) prices of the branded product rise and this increase in price more than offsets the loss of customers to the entrants.

A.1 Pre-Entry Equilibrium

Proposition 1. *There is a Nash equilibrium where firms* 0 *and* 1 *both set a price of* 1*. Each firm sells to the half of the market nearest to its position. Each firm makes a profit of* 0.5*, so that the combined profit is* 1*.*

Proof. If firm 1 sets a price of 1, then firm 0's profit when it sets a price $0 \leq p \leq 1$ is given by:

$$\pi_0 = p \times [0.5 + 0.5(1 - p)] = p(1 - 0.5p).$$

If firm 0 sets $p = 0$, it will sell to the whole market. If it sets $p = 2$, it sells to no customers. Its profit function π_0 is maximized by setting $p = 1$. A parallel argument shows that, if firm 0 sets a price of 1, firm 1 maximizes profit by setting $p = 1$.

Turning to the customers, note that, since all customers have a gross willingness-to-pay of at least 2 and "transportation costs" are equal to or lower that 0.5, all customers prefer to buy the branded product at a price $p = 1$, to not buying it. Indeed, this is true for the customer located at $x = 0.5$ and therefore it is true for all customers. □

A.2 Post-Entry Equilibrium

Proposition 2. *There is a Nash equilibrium in which the two firms offering the unbranded product each set a price of* 0*, and the two firms offering the branded product each set a price of* 1.98*. Each of the latter two firms makes a profit of* 0.594*, so that their combined profit is* 1.188*.*

Proof. The two firms offering the unbranded product are engaged in Bertrand competition and therefore each sets a price of 0. Turning to the firms offering branded products, if firm 1 charges a price of 1.98, then

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firm 0's profit function is given by:

$$\pi_0 = \begin{cases} p \times [0.3 - 0.5(p - 1.98)] & \text{if } 1.98 \leq p \leq 2.58; \\ p \times 0.3 & \text{if } 1.58 \leq p < 1.98; \\ p \times [0.3 + 0.5(1.58 - p)] & \text{if } 0.98 \leq p < 1.58; \\ p \times [0.6 + 0.5(0.98 - p)] & \text{if } 0.58 \leq p < 0.98; \\ p \times 1 & \text{if } 0 \leq p < 0.58. \end{cases}$$

If firm 0 raise its price above 1.98, it loses customers to the unbranded product. To see this, note that the customer located just below $x = 0.3$ nets $0.92 - 0.2 = 0.72$ if buying from an unbranded firm and nets $3 - 0.3 - 1.98 = 0.72$ if buying from firm 0. The first-order condition for firm 0 is:

$$\frac{d\pi_0}{dp} = (0.3 + 0.5 \times 1.98) - p,$$

which is negative for $p \geq 1.98$. Therefore, in this range, firm 0 maximizes profit by setting $p = 1.98$. Its profit is $0.3 \times 1.98 = 0.594$.

In the next interval, firm 0 does not gain any customers of the unbranded product. For this to happen, it would need to lower price to 0.98, in order to attract the customer of the unbranded product located at $x = 0.3$. To attract customers from firm 1, firm 0 needs to undercut by 0.4 and thereby attract the customer located at $x = 0.7$. So, firm 0 does not do better by lowering its price to a level within the second interval.

If firm 0 does choose to attract customers from firm 1, it maximizes profit by setting $p = 0.3 + 0.5 \times 1.58 = 1.09$. Its profit is 0.594. It will be part of an equilibrium strategy for firm 0 to choose the higher price $p = 1.98$.

The remaining cases are when firm 0 sets a sufficiently low price to attract some customers of the unbranded product. Then, firm 0 will have attracted all of firm 1's customers. If firm 0 sets a price above 0.58, it will only attract customers at locations $x \leq 0.5$. In this range, the first-order condition for firm 0 is:

$$\frac{d\pi_0}{dp} = (0.6 + 0.5 \times 1.98) - p,$$

which is positive. If firm 0 sets a price below 0.58, it will attract all the customers located at $x \in [0.5, 0.7]$. Its profit is 0.58.

We have established that if firm 1 sets a price of 1.98, then it is a best response for firm 0 to set a price 1.98. Individual firm profits – and therefore combined profit – are higher than in the pre-entry game. □

The idea of this game is that the entry of the unbranded substitute shifts more price-sensitive customers to the firms that offer this product. The unbranded firms take 40 percent of the market, but the incumbents undertake a nearly 2x increase in price, and the net effect is to raise their profits.