Symmetry and the Sixth Force: The Essential Role of Complements Online Appendix*

Adam Brandenburger † Barry Nalebuff ‡

10/05/26

This section presents a formal model to support our Example 1 in Section 3 of the main text. Customers, of total mass one, are uniformly located on the line segment $[0, 1]$. There are two incumbent firms. Firm 0 is located at 0, and firm 1 is located at 1. Initially, only branded products are available in the market. The customer located at x has a willingness-to-pay of $v(x) - x$ for the branded product from firm 0 and a willingness-to-pay of $v(x) - |1 - x|$ for the branded product from firm 1. We assume that, for all customers $x \in [0, 0.3)$ and $x \in (0.7, 1]$, the gross willingness-to-pay is 3. For all customers $x \in [0.3, 0.7]$, the gross willingness-to-pay is 2.

When an unbranded substitute arrives in the market, two firms, both located at 0.5, offer identical versions of this product. The customer located at x has a willingness-to-pay of $w(x) - |x - 0.5|$, where $w(x) = 0.93$ for all customers.

All firms have zero costs and no capacity constraints. In each game, the firms simultaneously choose nonnegative prices, and customers then make their purchase decisions. Each customer buys at most one unit, choosing a product that maximizes willingness-to-pay minus price, or choosing not to buy, which gives a payoff of zero. A firm's profit is its price times the mass of customers buying from it. Fix any measurable rule for resolving ties between payoff-maximizing choices. The equilibrium prices and profits established below do not depend on this rule.

The idea of the model is that, when the unbranded substitute enters the market, it captures all the price-sensitive customers in the "middle" of the market. The remaining customers at the "ends" of the market have a strong preference for one or other branded product. We will see that the (equal) prices of the branded products rise and this increase in price more than offsets the loss of customers to the entrants.

1 Pre-Entry Equilibrium

Proposition 1. *There is a Nash equilibrium where firms* 0 *and* 1 *both set a price of* 1*. Each firm sells to the half of the market nearest to its position. Each firm makes a profit of* 0.5*, so that the combined profit is* 1*.*

Proof. Suppose that firm 1 sets a price of 1, and let $p \geq 0$ be firm 0's price. Since $v(x) - (1 - x) - 1 > 0$ for every $x \in [0, 1]$, every customer can obtain a strictly positive payoff by buying from firm 1. Thus no customer chooses not to buy. A customer weakly prefers firm 0 to firm 1 when:

$$v(x) - x - p \geq v(x) - (1 - x) - 1,$$

or, equivalently:

$$x \leq 1 - \frac{p}{2}.$$

Any indifferent customer is a single point and has zero mass. Therefore, firm 0's profit is:

$$\pi_0(p) = \begin{cases} p\left(1 - \frac{p}{2}\right), & 0 \leq p \leq 2, \\ 0, & p > 2. \end{cases}$$

On $[0, 2]$, its derivative is $1 - p$, so profit is uniquely maximized at $p = 1$, yielding 0.5. By symmetry, if firm 0 charges 1, firm 1's unique best-response price is also 1.

*Important input from Chloe Chen, Judy Chevalier, Jessy Hsieh, and Natalya Vinokurova is gratefully acknowledged. We thank the NYU Stern School of Business, NYU Shanghai, and J.P. Valles for financial support.

†Stern School of Business, Tandon School of Engineering, NYU Shanghai, New York University, New York, NY 10012, U.S.A., adam.brandenburger@stern.nyu.edu

‡Yale School of Management, New Haven, CT 06511, U.S.A., barry.nalebuff@yale.edu

At these prices, customers buy from the nearer firm, with the customer at $x = 0.5$ indifferent between the two. Each firm therefore serves half the market and earns 0.5. □

2 Post-Entry Equilibrium

Proposition 2. *There is a Nash equilibrium in which the two firms offering the unbranded product each set a price of* 0*, and the two firms offering the branded product each set a price of* 1.97*. Each branded firm serves 30 percent of the market and makes a profit of* 0.591*, so that their combined profit is* 1.182*.*

Proof. The two firms offering the unbranded product are engaged in Bertrand competition and therefore each sets a price of 0. Suppose that both branded firms charge 1.97. A customer at $x \in [0, 0.3)$ obtains:

$$3 - x - 1.97 = 1.03 - x,$$

from firm 0, compared with:

$$0.93 - (0.5 - x) = 0.43 + x,$$

from an unbranded firm. The difference is $0.6 - 2x > 0$. The customer also prefers firm 0 to firm 1, since the branded prices are equal and firm 0 is nearer. Therefore, all customers in $[0, 0.3)$ buy from firm 0. By symmetry, all customers in $(0.7, 1]$ buy from firm 1.

For a customer in $[0.3, 0.7]$, the highest payoff from a branded product is:

$$2 - \min\{x, 1 - x\} - 1.97 = 0.03 - \min\{x, 1 - x\} \leq -0.27.$$

An unbranded product instead gives a payoff of at least $0.93 - 0.2 = 0.73$. Therefore, all customers in the middle segment buy an unbranded product. Each branded firm sells to a mass 0.3 of customers and earns $0.3 \times 1.97 = 0.591$.

It remains to check all unilateral price deviations by a branded firm. Fix firm 1's price at 1.97 and both unbranded prices at 0, and let $p \geq 0$ be firm 0's price. The unbranded product gives every customer a strictly positive payoff, so the outside option is irrelevant for these deviations. A customer weakly prefers firm 0 to all its alternatives when:

$$v(x) - x - p \geq \max\big\{v(x) - (1 - x) - 1.97,\ 0.93 - |x - 0.5|\big\}.$$

Comparing these payoffs on each customer segment gives the following profit function for $p \neq 0.57$:

$$\pi_0(p) = \begin{cases} 0, & p > 2.57, \\ p\big[0.3 - 0.5(p - 1.97)\big], & 1.97 \leq p \leq 2.57, \\ 0.3p, & 1.57 \leq p < 1.97, \\ p\big[0.3 + 0.5(1.57 - p)\big], & 0.97 \leq p < 1.57, \\ p\big[0.6 + 0.5(0.97 - p)\big], & 0.57 < p < 0.97, \\ p, & 0 \leq p < 0.57. \end{cases}$$

At $p = 0.57$, customers in $[0.5, 0.7]$ are indifferent between firm 0 and an unbranded firm, since:

$$2 - x - 0.57 = 0.93 - (x - 0.5) = 1.43 - x.$$

Regardless of how these ties are resolved, firm 0's sales cannot exceed the total market size of 1, so that:

$$\pi_0(0.57) \leq 0.57 < 0.591.$$

Consider first $1.97 \leq p \leq 2.57$. Increasing the price above 1.97 loses customers to the unbranded product. The derivative is:

$$\frac{d\pi_0}{dp} = 0.3 + 0.5 \times 1.97 - p = 1.285 - p < 0.$$

Profit is therefore maximized on this interval at $p = 1.97$, where it is 0.591. For $p > 2.57$, profit is 0.

For $1.57 \leq p < 1.97$, firm 0 retains its original customers but gains no positive mass of additional customers. To attract a positive mass from firm 1, it must lower its price below 1.57, undercutting firm 1 by more than 0.4 and attracting customers just to the right of $x = 0.7$. To attract a positive mass from the unbranded firms, it must lower its price below 0.97. Therefore, profit throughout $1.57 \leq p < 1.97$ is $0.3p < 0.591$.

For $0.97 \leq p < 1.57$, firm 0 attracts some or all of firm 1's customers but no positive mass of customers from the unbranded firms. Its profit is:

$$\pi_0(p) = p\big[0.3 + 0.5(1.57 - p)\big] = p(1.085 - 0.5p).$$

This expression is uniquely maximized within the interval at $p = 1.085$. The resulting profit is:

$$\pi_0(1.085) = 1.085 \times 0.5425 = 0.5886125 < 0.591.$$

Therefore, even the best deviation in this interval is strictly less profitable than charging 1.97.

For $0.57 < p < 0.97$, firm 0 has already attracted all of firm 1's customers and additionally attracts some customers from the unbranded firms. These additional customers are located to the left of 0.5. The derivative of profit is:

$$\frac{d\pi_0}{dp} = 0.6 + 0.5 \times 0.97 - p = 1.085 - p > 0.$$

Profit throughout this interval is therefore strictly below its limiting value at $p = 0.97$, which is:

$$0.97 \times 0.6 = 0.582 < 0.591.$$

For $0 \leq p < 0.57$, firm 0 attracts the whole market, but its profit is $p < 0.57 < 0.591$. The price $p = 0.57$ was covered by the bound above.

These comparisons cover every nonnegative price and show that firm 0's unique best-response price is 1.97. The symmetric argument establishes the same result for firm 1. Together with the unbranded firms' best responses, this proves the claimed Nash equilibrium. Combined incumbent profit is $2 \times 0.591 = 1.182 > 1$. □

The idea of this game is that the entry of the unbranded substitute shifts more price-sensitive customers to the firms that offer this product. The unbranded firms take 40 percent of the market, but the incumbents undertake a nearly 2x increase in price, and the net effect is to raise their profits.