Introduction to Game Theory:

Two-by-Two Games

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If $C > B > C/2 > 0$, what game is this?

<table>
<thead>
<tr>
<th>Inspect predator</th>
<th>Don’t inspect predator</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B - C/2$</td>
<td>$B$</td>
</tr>
<tr>
<td>$B - C/2$</td>
<td>$B - C$</td>
</tr>
<tr>
<td>$B - C$</td>
<td>$0$</td>
</tr>
<tr>
<td>$B$</td>
<td>$0$</td>
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</tbody>
</table>
General Two-By-Two Matrix

1. Payoffs are player-specific evaluations of outcomes.
2. What if payoffs are not ‘transparent’ to the players? Later ...
The Number of Two-By-Two Matrices

How many $2 \times 2$ matrix games are there?

Let’s simplify the question:

Assume there are no ties among $a, b, c, d$, and no ties among $\alpha, \beta, \gamma, \delta$

So, Ann (resp. Bob) has a strict ranking of $a, b, c, d$ (resp. $\alpha, \beta, \gamma, \delta$)

How many rankings of $a, b, c, d$ (resp. $\alpha, \beta, \gamma, \delta$) are there?
The Number of Two-By-Two Matrices Cont’d

If we distinguish games via the players’ ordinal rankings of payoffs, how many different $2 \times 2$ matrix games are there?

We can consider as strategically equivalent any two matrices where one can be obtained from the other by: (1) interchanging rows, (2) interchanging columns, (3) interchanging players, (4) any sequence of these operations

After this reduction, we would obtain 78 strategically distinct $2 \times 2$ matrix games*

This is still too many to remember!

Let’s take a more heuristic approach ...
Symmetric Two-By-Two Matrices

We will rank only $a$ with respect to $c$, and $b$ with respect to $d$

In this scheme, how many distinct symmetric $2 \times 2$ matrix games are there?
We assume that $a > d$ (this is just a choice of orientation)
What conceptually new behavior of the arrows arises in the asymmetric case?
A Scheme of Two Asymmetric Two-By-Two Matrices

Matching Pennies

One-Sided Game
Preview of Analysis of Game Matrices

In the Prisoner’s Dilemma, the strategy $X$ for Ann is dominant (also, undominated) and the strategy $Y$ is dominated; and likewise for Bob.

In the Battle of the Sexes and the Coordination Game, there are no dominance relationships.

In the Battle of the Sexes, the pairs of strategies $(X, Y)$ and $(Y, X)$ constitute Nash equilibria.

In the Coordination Game, the pairs of strategies $(X, X)$ and $(Y, Y)$ constitute Nash equilibria.

In Matching Pennies, there is no Nash equilibrium (in “pure” strategies).

In the One-Sided Game, the strategy $X$ for Ann is dominant and the strategy $Y$ for Bob is iteratively undominated.