

# **Introduction to Game Theory:**

**Levels of Reasoning**

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## How Many Levels Do Players Reason?

When we first hear about game theory, we are naturally led to ponder:

How does a player Ann in a game think not just about what moves another player Bob might make, but also about what Bob might be thinking about her own moves, and, perhaps, about still higher levels of thinking about thinking?

But we have seen that the historical development of game theory sidestepped this issue

In von Neumann's minimax theory players adopt a 'protective' rather than predictive view of what other players do, and choose accordingly

In Nash's equilibrium theory, each player is assumed to have access to the actual strategies chosen by the other players and to choose a strategy accordingly

## Iterated Dominance

It is in iterated dominance methods that game theory comes closest to addressing this issue

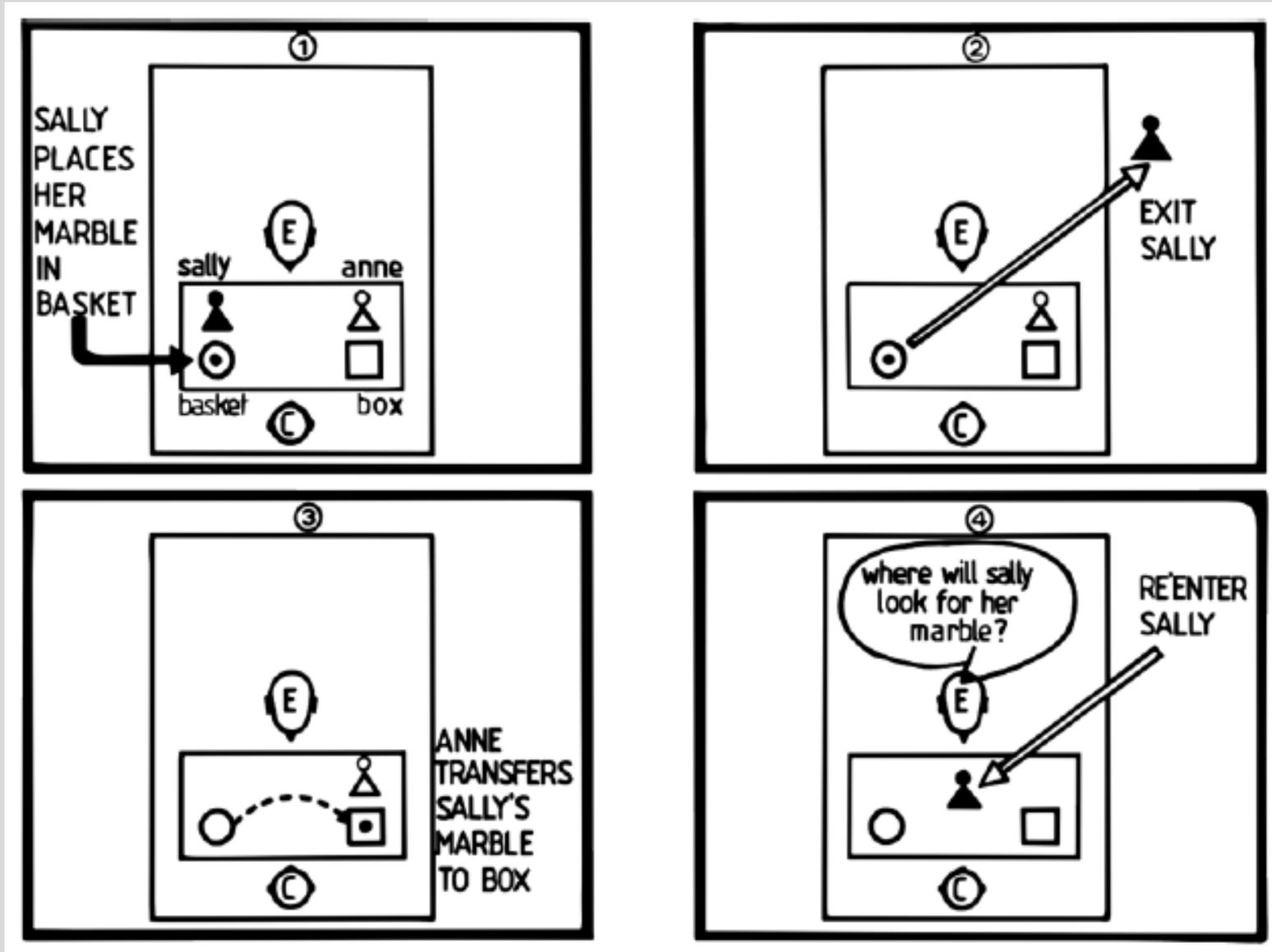
Still, it has been traditional to assume that players can engage in indefinitely many levels of reasoning --- corresponding to indefinitely many levels of iterated dominance

It seems this approach fails to take account of cognitive limitations to the human brain

Let's see what the cognitive sciences --- cognitive psychology and cognitive neuroscience --- can teach us

There is also relevant recent work in experimental game theory

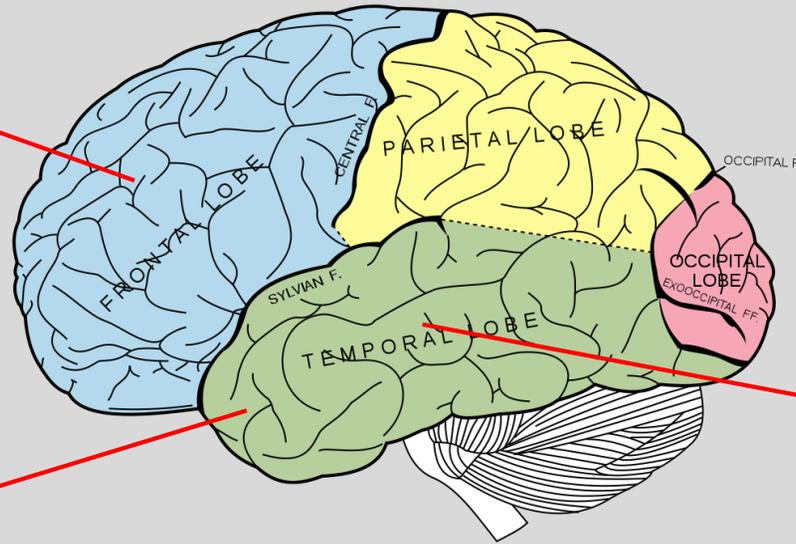
## Development of "Theory of Mind"



## Neuroscience of ToM

### Medial prefrontal cortex →

Locus of actual neural representation of mental states (intentions, desires, knowledge, and beliefs) of others. There may be an anchoring and adjustment process that begins with one's own mental states and then operates to try to infer the mental states of others.



### Temporal poles →

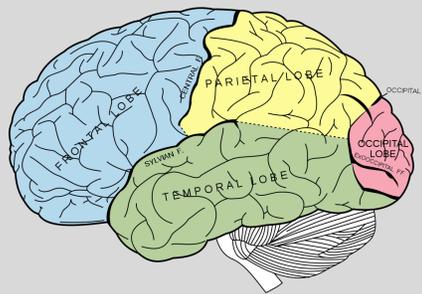
Activated in exercises involving language and memory retrieval. These regions may build up “scripts” that facilitate predicting how others will behave in the future.

### Posterior superior temporal sulcus →

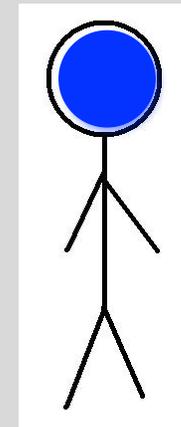
Activated on reception of inputs (motion, sound, light) about the (current) behavior of other living agents.

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# Activation of ToM in Games



or



McCabe, K. et al., "A Functional Imaging Study of Cooperation in Two-Person Reciprocal Exchange," *PNAS*, 98, 2001, 11832-11835; Gallagher, H., et al., "Imaging the Intentional Stance in a Competitive Game," *NeuroImage*, 16, 2002, 814-821; Rilling, J., et al., "The Neural Correlates of Theory of Mind Within Interpersonal Interactions," *NeuroImage*, 22, 2004, 1694-1703;  
<http://upload.wikimedia.org/wikipedia/commons/thumb/1/1a/Gray728.svg/1024px-Gray728.svg.png>

## ToM Ability

A person's ToM ability is usually defined via a task where the person hears a short story describing a social situation and is then asked questions about the story

The questions differ in terms of the number of levels of “Ann thinks Bob thinks Charlie thinks ...” that they contain (Ann, Bob, and Charlie are characters in the story)

The maximum number of such levels that a question can contain and still be answered correctly by the person gives that person's ToM ability

Recently, experimental game theorists have also measured ToM ability

## Examples of Narratives

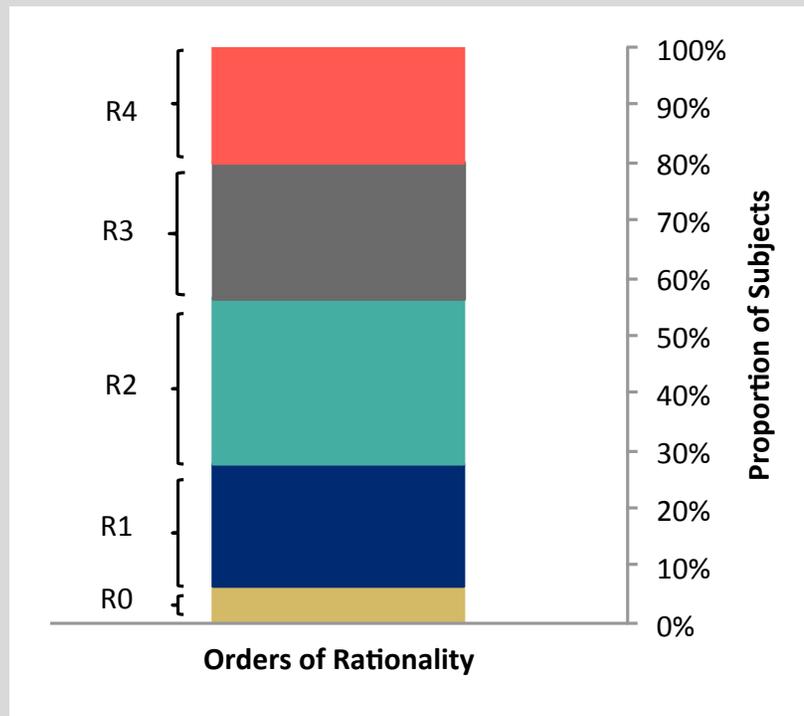
A burglar who has just robbed a shop is making his getaway. As he is running home, a policeman on his beat sees him drop his glove. He doesn't know the man is a burglar, he just wants to tell him he dropped his glove. But when the policeman shouts out to the burglar, "Hey, you! Stop!", the burglar turns round, see the policeman and gives himself up. He puts up his hands and admits that he did the break-in at the local shop. **Question:** Why did the burglar do that? (**ToM story**)

Two enemy powers have been at war for a very long time. Each army has won several battles, but now the outcome could go either way. The forces are equally matched. However, the Blue army is stronger than the Yellow army in foot soldiers and artillery. But the Yellow army is stronger than the Blue army in air power. On the day of the final battle, which will decide the outcome of the war, there is heavy fog over the mountains where the fighting is about to occur. Low-lying clouds hang above the soldiers. By the end of the day the Blue army has won. **Question:** Why did the Blue army win? (**Non-ToM story**)

# Findings

Narrative (Stiller and Dunbar, 2007)	Modal level of failure is 5 (-1?)*
Game (Arad and Rubinstein, 2012)	Maximum level is 3
Game (Kneeland, 2015)	Maximum level is 4 (see below)

Percentage of players: 22  
22  
27  
22  
7



\* Women achieved one more level (an average of 5.53 levels) than did men (an average of 4.41 levels), a difference which was significant; Stiller, J., and R. Dunbar, "Perspective-Taking and Memory Capacity Predict Social Network Size," *Social Networks*, 29, 2007, 93-104; Arad, A., and A. Rubinstein, "The 11-20 Money Request Game: A Level-*k* Reasoning Study," *American Economic Review*, 102, 2012, 3561-3573; Kneeland, T., "Identifying Higher-Order Rationality," *Econometrica*, 83, 2015, 2065-2079; diagram used with permission

## Open Issues

### 1. Identifying cognition vs. rationality?

Cognition refers to the process of thinking, while rationality can be viewed as a process of thoughtfully and well-chosen (e.g., undominated) action

Then, we can have cognition without rationality, although we can have rationality without cognition

This means we have to be careful drawing inferences from game experiments:

Suppose Ann chooses a strategy which survives one but not two rounds of iterated dominance. How many levels does Ann reason? Perhaps two, if she thinks Bob is thinking about the game, but she notices he chose a dominated strategy in a previous game \*

### 2. Cognitive mechanism?

If the number of possibilities that a player must consider at each level  $m$  of thinking grows exponentially with  $m$ , we can expect a cognitive bound at a small finite number  $m$  \*\*

\* Friedenberg, A., W. Kets, and T. Kneeland, "Cognition and Rationality," 2016, at [amandafriedenberg.org](http://amandafriedenberg.org);  
Jin, Y., "Does Level- $k$  Behavior Imply Level- $k$  Thinking?" 2016, at <https://sites.google.com/site/yjinecon/>;

\*\* Brandenburger, A., and X. Li, "Thinking About Thinking and Its Cognitive Limits," 2015, at <http://adambrandenburger.com/articles/papers/>

## Appendix: The Social Brain Hypothesis



## Appendix: Group Size vs. Brain Size

