

Introduction to Game Theory:

Two-By-Two Games

NYU Shanghai

Spring 2017

Sticklebacks



*Inspect
predator*

*Don't inspect
predator*

*Inspect
predator*

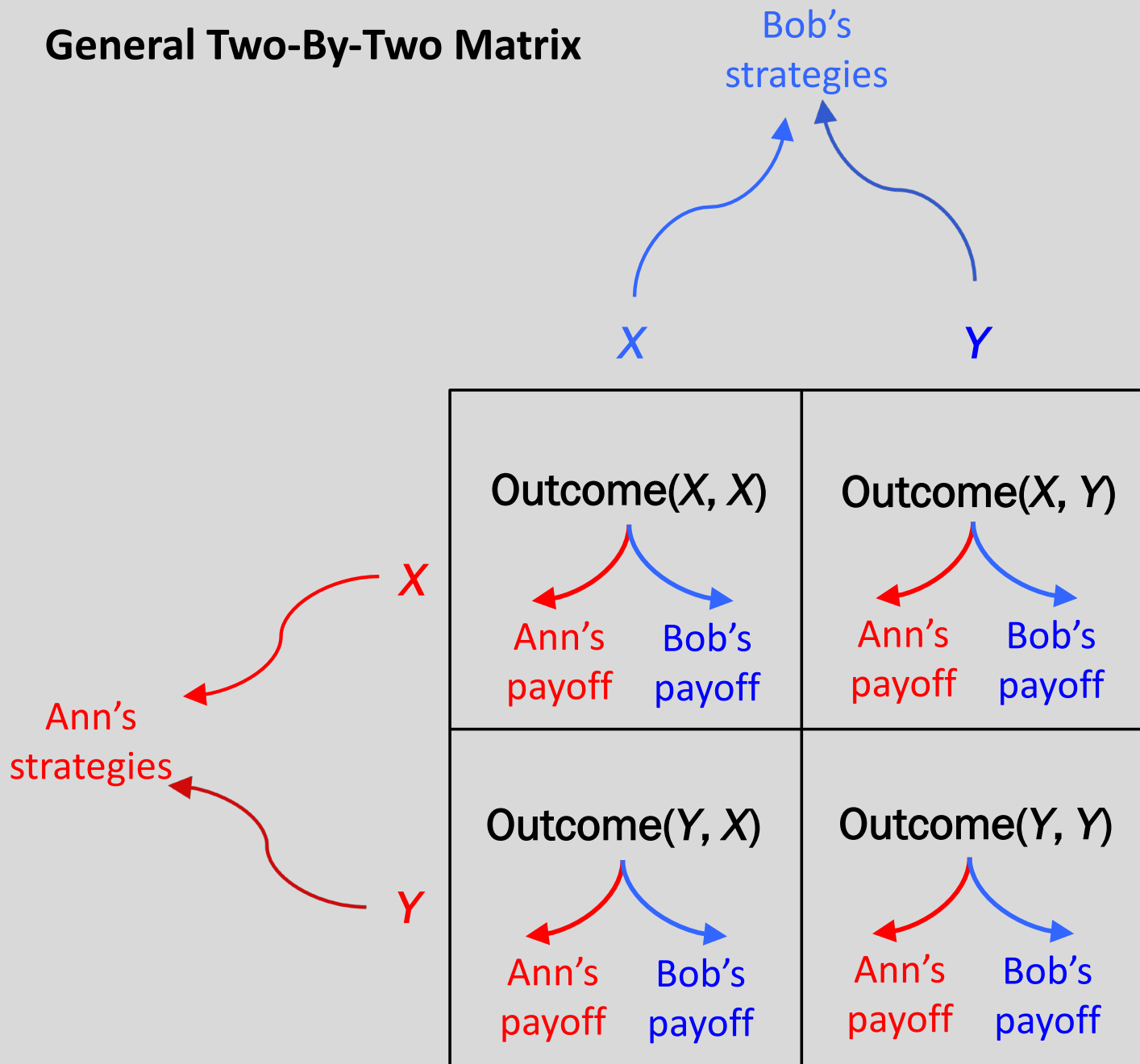


*Don't inspect
predator*

	<i>Inspect predator</i>	<i>Don't inspect predator</i>
<i>Inspect predator</i>	$B - C/2$ $B - C/2$	B $B - C$
<i>Don't inspect predator</i>	$B - C$ B	0 0

If $C > B > C/2 > 0$, what game is this?

General Two-By-Two Matrix



1. Payoffs are player-specific evaluations of outcomes
2. What if payoffs are not 'transparent' to the players?
Later ...

The Number of Two-By-Two Matrices

How many 2×2 matrix games are there?

		Bob	
		γ	χ
Ann	Y	α a	β b
	X	γ c	δ d

Let's simplify the question:

Assume there are no ties among a, b, c, d , and no ties among $\alpha, \beta, \gamma, \delta$

So, Ann (resp. Bob) has a strict ranking of a, b, c, d (resp. $\alpha, \beta, \gamma, \delta$)

How many rankings of a, b, c, d (resp. $\alpha, \beta, \gamma, \delta$) are there?

The Number of Two-By-Two Matrices Cont'd

If we distinguish games via the players' ordinal rankings of payoffs, how many different 2×2 matrix games are there?

We can consider as strategically equivalent any two matrices where one can be obtained from the other by: (1) interchanging rows, (2) interchanging columns, (3) interchanging players, (4) any sequence of these operations

After this reduction, we would obtain 78 strategically distinct 2×2 matrix games*

This is still too many to remember!

Let's take a more heuristic approach ...

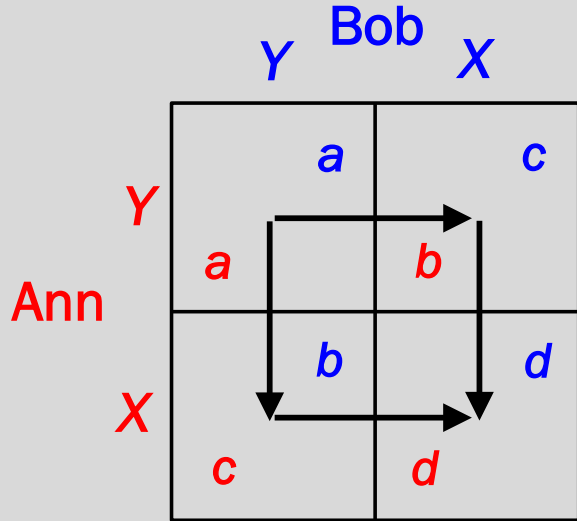
Symmetric Two-By-Two Matrices

		Y	Bob	X
			a	c
Y	Ann	a		b
			b	d
X		c		d

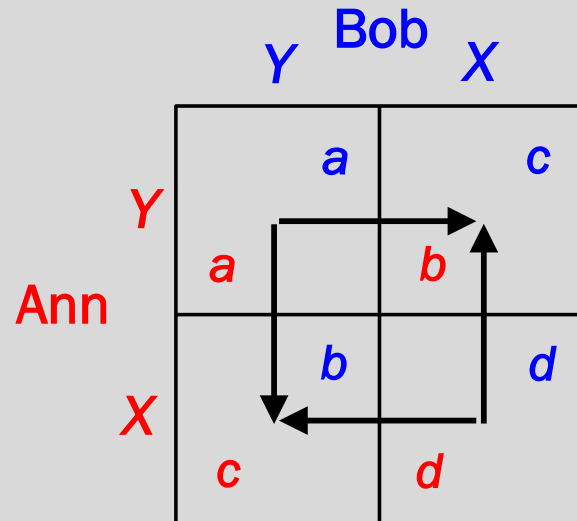
We will rank only a with respect to c , and b with respect to d

In this scheme, how many distinct symmetric 2×2 matrix games are there?

A Scheme of Four Symmetric Two-By-Two Matrices

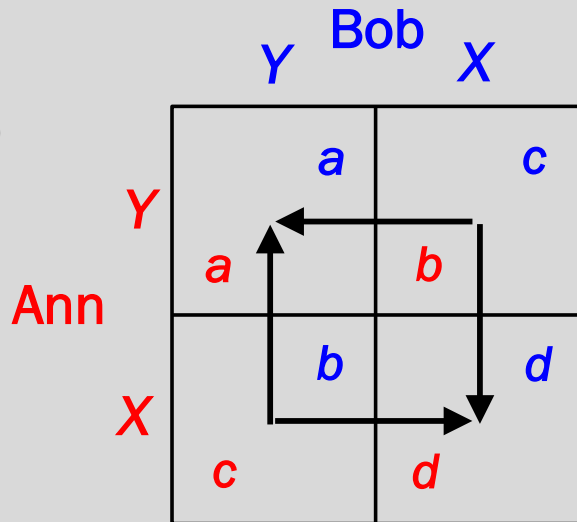


Prisoner's Dilemma

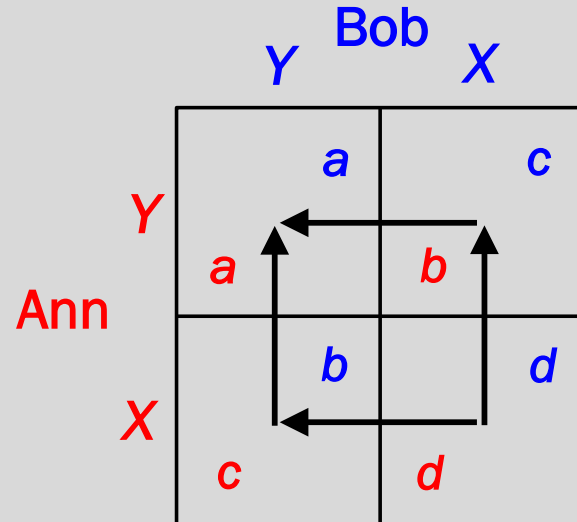


Battle of the Sexes

We assume that $a > d$ (this is just a choice of orientation)



Coordination Game



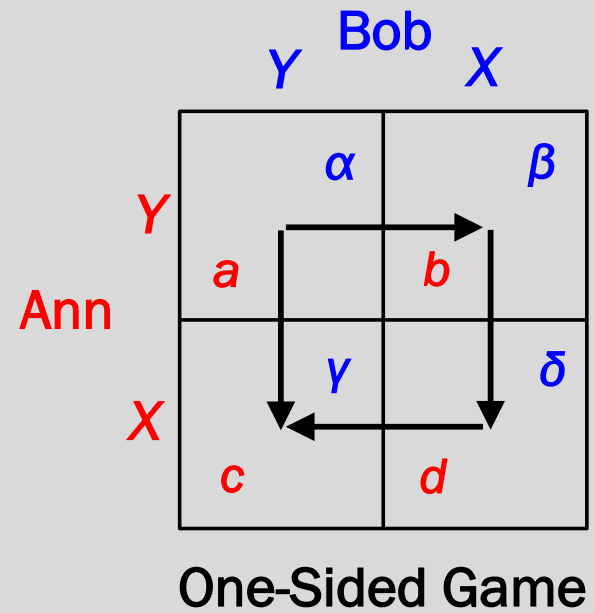
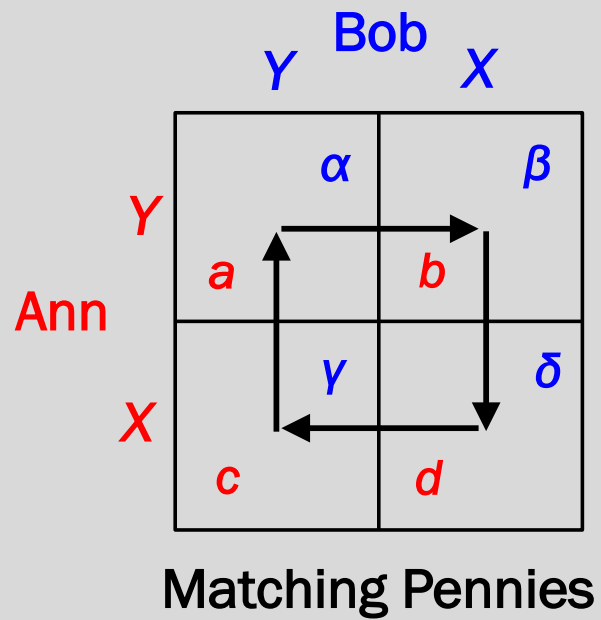
Prisoner's Delight

Asymmetric Two-By-Two Matrices

	γ	Bob	χ
Y	α		β
Ann	a		b
X	γ		δ
	c		d

What conceptually new behavior of the arrows arises in the asymmetric case?

A Scheme of Two Asymmetric Two-By-Two Matrices



Preview of Analysis of Game Matrices

In Prisoner's Dilemma, the strategy X for Ann is **dominant** (also, undominated) and the strategy Y is **dominated**; and likewise for Bob

In Battle of the Sexes and Coordination Game, there are no dominance relationships

In Battle of the Sexes, the pairs of strategies (X, Y) and (Y, X) constitute **Nash equilibria**

In Coordination Game, the pairs of strategies (X, X) and (Y, Y) constitute **Nash equilibria**

In Matching Pennies, there is no Nash equilibrium (in "pure" strategies)

In One-Sided Game, the strategy X for Ann is dominant and the strategy Y for Bob is **iteratively undominated**