The Bigger Picture . . . or the Smaller Picture

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1 Introduction

In his classic book *How to Solve It*,[1] the mathematician George Pólya offered advice on how to tackle problems in mathematics. Among his advice is:

> If you cannot solve the proposed problem, try to solve first some related problem. Could you imagine a more accessible related problem? A more general problem? A more special problem? An analogous problem? Could you solve a part of the problem?

In this note we offer some brief thoughts on two components of this advice, one about looking to break off a part of the problem at hand and solve the smaller problem, the other about looking to embed the problem at hand in a bigger problem.

2 The Smaller Picture

There are problems that cannot be solved even in principle, regardless of the resources, intellectual or material, that are devoted to them. In 1937, the computer pioneer Alan Turing found a famous such problem,[2] when he proved that there cannot exist an algorithm that, given any computer program together with an input, can decide whether or not the program will halt on that input.

Then there are problems which could in principle be solved, but the solutions of which lie beyond today's technological capabilities. The game of Chess makes a good example.[3] It is straightforward to prove that for Chess, precisely one of the following three statements is true: (i) there is a strategy for White that guarantees White a win every time, regardless of how Black plays; (ii) there is a strategy for Black that guarantees Black a win every time, regardless of how White plays; (iii) there is a strategy for White that guarantees White at least a draw, and the same is true for Black.[4] In principle, it is possible to determine which of these three statements is true, and even to determine the corresponding strategy or strategies, by drawing out the game tree for Chess and working backwards through the tree. But this is only in principle. The reason is that the tree is so large — estimated to contain more than 10^{120} different paths through it[5] — that no conceivable machine could analyze it. In practice, Chess is (partially) analyzed by being broken down into parts usually refereed to as the opening, middle game, and endgame. Many endgames, in particular, have been analyzed exhaustively and complete solutions are known for many of them.[6]

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So far, the problems we have mentioned lie in the mathematical realm. For problems in the social domain, too, it is sometimes argued that the best way to proceed is by trying to solve a smaller problem. Economist Elinor Ostrom argued this way in the case of common-resource problems. The starting point of her work was the conventional wisdom of the "tragedy of the commons" — that common resources such as fisheries or forests will be over-exploited and the resource thereby destroyed.[7] Field studies around the world combined with experimental work led Ostrom to conclude that common resources were preserved more often than the tragedy-of-the-commons view indicated.[8] She listed a number of factors that appear to favor behavior by users that preserves common resources:[9]

(i) User Boundaries: Clear and locally understood boundaries between legitimate users and nonusers are present.

(ii) Resource Boundaries: Clear boundaries that separate a specific common-pool resource from a larger social-ecological system are present.

(iii) Congruence with Local Conditions: Appropriation and provision rules are congruent with local social and environmental conditions.

(iv) Appropriation and Provision: Appropriation rules are congruent with provision rules; the distribution of costs is proportional to the distribution of benefits.

(v) Collective Choice Arrangements: Most individuals affected by a resource regime are authorized to participate in making and modifying its rules.

(vi) Monitoring Users: Individuals who are accountable to or are the users monitor the appropriation and provision levels of the users.

(vii) Monitoring the Resource: Individuals who are accountable to or are the users monitor the condition of the resource.

(viii) Graduated Sanctions: Sanctions for rule violations start very low but become stronger if a user repeatedly violates a rule.

(ix) Conflict Resolution Mechanisms: Rapid, low cost, local arenas exist for resolving conflicts among users or with officials.

(x) Minimal Recognition of Rights: The rights of local users to make their own rules are recognized by the government.

(xi) Nested Enterprises: When a common-pool resource is closely connected to a larger social-ecological system, governance activities are organized in multiple nested layers.

Of particular interest in the context of this note is the emphasis on a local perspective on common-resource systems. Even though many problems of sustainability are often considered at a global — even planetary — level, Ostrom's work suggests that such problems may be usefully addressed by being broken down into smaller local problems to be tackled at the local level. Here is a summary of her approach:[10]

> An ever larger number of researchers — including Elinor Ostrom, the 2009 Nobel Prize winner in economics — has cautioned against single governmental units at global level to solve these global collective action problems, due in part to their complexity, and in part to the diversity of actors involved. Ostrom proposes a polycentric approach at various levels of society; with active oversight of local, regional and national stakeholders.

3 The Bigger Picture

We again begin with mathematics and then move to other domains. In 1900, the great mathematician David Hilbert gave a speech at the Second International Congress of Mathematicians in Paris, in which he described 23 open problems that prompted much mathematical research during the 20th century. (Turing's 1937 paper mentioned earlier solved one of the 23 problems.) Hilbert also spoke about techniques for solving problems — both about specialization and about generalization. On the latter he said:[11]

> If we do not succeed in solving a mathematical problem, the reason frequently consists in our failure to recognize the more general standpoint from which the problem before us appears only as a single link in a chain of related problems. After finding this standpoint, not only is this problem frequently more accessible to our investigation, but at the same time we come into possession of a method which is applicable also to related problems.

A highly influential concept, in a very different domain, that urges us to take a bigger perspective is that of "marketing myopia," introduced by business-school professor Ted Levitt in 1960. Levitt argued that businesses often take a narrow 'inside-out' view of what they do rather than a broader 'outside-in' view. Businesses can spend too much time on the problem of managing their own operations, and not enough time on the bigger problem of what customers really need or want in the first place. He wrote provocatively:[12]

> Every major industry was once a growth industry. But some that are now riding a wave of growth enthusiasm are very much in the shadow of decline. Others that are thought of as seasoned growth industries have actually stopped growing. In every case, the reason growth is threatened, slowed, or stopped is not because the market is saturated. It is because there has been a failure of management .... The failure is at the top. The executives responsible for it, in the last analysis, are those who deal with broad aims and policies. Thus: The railroads did not stop growing because the need for passenger and freight transportation declined. That grew. The railroads are in trouble today not because that need was filled by others (cars, trucks, airplanes, and even telephones) but because it was not filled by the railroads themselves. They let others take customers away from them because they assumed themselves to be in the railroad business rather than in the transportation business. The reason they defined their industry incorrectly was that they were railroad oriented instead of transportation oriented; they were product oriented instead of customer oriented.

Exercises:

- Give an instance of a former or current real-world problem that was or, you claim, is usefully addressed by extracting a part of it and addressing that part.
- Give an instance of a former or current real-world problem that was or, you claim, is usefully addressed by embedding it in a larger problem and addressing that problem.

Notes

[1]Princeton University Press, 1945.

[2]Turing, A., "On Computable Numbers, with an Application to the Entscheidungsproblem," *Proceedings of the London Mathematical Society*, Series 2, 42, 1937, 230-265.

[3]We suppose that there is a stopping rule in force, such as the threefold repetition rule, to prevent indefinite play.

[4]Clearly, at most one of these three statements can be true. The force of the assertion is that one of them is indeed true. This is (incorrectly) often referred to as Zermelo's Theorem. For the correct history of this result, see Schwalbe, U., and P. Walker,"Zermelo and the Early History of Game Theory," *Games and Economic Behavior*, 34, 2001, 123-137.

[5]The first such estimate is in Shannon, C., "Programming a Computer for Playing Chess," *Philosophical Magazine*, 41, 1950, 314.

[6]See wikipedia.org/wiki/Endgame_tablebase.

[7]Hardin, G., "The Tragedy of the Commons, *Science*, 162, 1968, 1243-1248.

[8]Ostrom, E., *Governing the Commons: The Evolution of Institutions for Collective Action*, Cambridge University Press, 1990.

[9]This list is from Ostrom, E., "Beyond Markets and States: Polycentric Governance of Complex Economic Systems," *American Economic Review*, 100, 2010, 1-33. It is presented as a version of the list in Cox M., G. Arnold, and S. Villamayor-Tomás, "A Review of Design Principles for Community-based Natural Resource Management," *Ecology & Society*, 15, 2010, special section 1-19. This latter is an evaluation and extension of Ostrom (1990).

[10]Vedeld, T., "A New Global Game — And How Best to Play It," at http://blog.nibrinternational.no/#post9.

[11]Hilbert, D., "Mathematical Problems," *Bulletin of the American Mathematical Society*, 8, 1902, 437-479. The 21st century analogs are the Clay Mathematics Institute Millennium Prize Problems which were announced in 2000; see http://www.claymath.org/millennium-problems.

[12]Levitt, T., "Marketing Myopia," *Harvard Business Review*, July-August, 1960.