Origins of Epistemic Game Theory

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The Origin?

> "[T]here is exhibited an endless chain of reciprocally conjectural reactions and counter-reactions. . . . The remedy would lie in analogous employment of the so-called Russell theory of types in logistics. This would mean that on the basis of the assumed knowledge by the economic subjects of theoretical tenets of Type I, there can be formulated higher propositions of the theory; thus, at least, of Type II. On the basis of information about tenets of Type II, propositions of Type III, at least, may be set up, etc."

– "Perfect Foresight and Economic Equilibrium," by Oskar Morgenstern, *Zeitschrift für Nationalökonomie*, 1935

Von Neumann's 1928 Paper

1. The concept of **strategy**
   "[I]t is possible to bring all games ... into a much simpler normal form .... Each player S_m ($m = 1, 2, \ldots, n$) chooses a number $1, 2, \ldots, N_m$ without knowing the choices of the others."

2. The **Minimax Theorem**
   "[H]e is protected against his adversary 'finding him out.'"

3. The concept of a **cooperative** game
   "[T]he three-person game is essentially different from a game between two persons.... It is [now] a question of which of the three equally possible coalitions $S_1, S_2; S_1, S_3; S_2, S_3$ has been formed. A new element enters, which is entirely foreign to the stereotyped and well-balanced two-person game: struggle."

An Influence on von Neumann?

> "The magnitude of the work that a group of [players] can perform under all varying possible conditions that may present themselves ... is an index of the ... value of that group."

–*Struggle*, by Emanuel Lasker, Lasker's Publishing Company, New York, 1907, p.31 (Lasker was World Chess Champion from 1897 to 1921)

Further reading: "New Light on von Neumann: Politics, Psychology and the Creation of Game Theory," by Robert Leonard, at www.cesmep.unito.it/WP/2007/7_WP_Cesmep.pdf.

Theory of Games and Economic Behavior

1. Maximin obviates epistemics

   "Nor are our results for one player based upon any belief in the rational conduct of the other." (p.160)

2. Indeterminism

   "[W]e shall in most cases observe a multiplicity of solutions. Considering what we have said about interpreting solutions as stable 'standards of behavior' this has a simple and not unreasonable meaning, namely that given the same physical background different 'established orders of society' or 'accepted standards of behavior' can be built. . . ." (p.42)

   a. Outcomes are under-determined by the game model.
   b. Additional factors–of a more 'intangible' kind–also matter.

The Equilibrium Criterion

Nash's reformulation (doctoral dissertation, 1950) removes the cooperative and maximin aspects and asks instead what is rational individual play.

> "We proceed by investigating the question: what would be a 'rational' prediction of the behavior to be expected of rational[ly] playing the game in question? By using the principles that a rational prediction should be unique, that the players should be able to deduce and make use of it, and that such knowledge on the part of each player of what to expect the others to do should not lead him to act out of conformity with the prediction, one is led to the concept of a solution defined before."

Steps in the argument:

1. Associated with each game is a unique correct way to analyze that game–cf. von Neumann-Morgenstern multiplicity.
2. This way is accessible to the players themselves–cf. distinguishing players and analyst/observer.
3. Each player makes the best choice of strategy for him/herself.

Von Neumann in 1955

> "Von Neumann pointed out that the enormous variety of solutions which may obtain for n-person games was not surprising in view of the correspondingly enormous variety of observed stable social structures; many differing conventions can endure, existing today for no better reason than that they were here yesterday."

–Report of an informal conference on *Recent Developments in the Theory of Games*, P. Wolfe (ed.), Department of Mathematics, Princeton University, 1955

Harold Kuhn and Robert Leonard kindly provided a copy of Wolfe (1955).

Types

Harsanyi (1967-8) wanted to analyze uncertainty about the structure of a game–specifically, about the players' payoff functions.

The Harsanyi formalism consists of, for each player i:

- a finite set T^i of **types** for player i;
- a map $f^i : T^i \rightarrow \mathcal{M}(T^{-i})$;
- a map $g^i : T^i \rightarrow S^i$ (if to $\mathcal{M}(S^i)$ then purify);
- a map $h^i : S \times T \rightarrow \mathbb{R}$.

At the true state—(U, t^a, R, t^b), say—we can calculate the players' hierarchies of beliefs over:

1. strategies,
2. rationality and irrationality.

Under the Bayesian-equilibrium approach (starting with Harsanyi's own numerical examples):

1. distinct types have distinct payoff functions, so there is no 'intrinsic' uncertainty about strategies,
2. all types optimize, so there is no irrationality.

Rationalizability and Beyond

Bernheim (1984) and Pearce (1984) made uncertainty about strategies central (but did not treat irrationality).

They argued informally that common knowledge of rationality is characterized by the **rationalizable** set (obtained by iteratively deleting strongly dominated strategies).

[The belief-knowledge distinction is very important elsewhere in EGT.]

EGT proper began with formal proofs of this assertion using type structures.

Subsequent topics in EGT:

a. irrationality,
b. epistemics of game trees,
c. epistemics of weak dominance,

...

On "common knowledge," see Aumann (1976), Lewis (1969), and Friedell (1967)–the last was re-discovered by Barry O'Neill.

Indeterminism Again

Where do the type structures of EGT come from?

Like payoffs, beliefs are subjective–neither can be deduced from the other components of a game.

> "We think of a particular ... structure as giving the 'context' in which the game is played. In line with Savage's Small-Worlds idea in decision theory (Foundations of Statistics, 1954), who the players are in the given game can be seen as a shorthand for their experiences before the game. The players' possible characteristics–including their possible types–then reflect the prior history or context. Each different type structure reflects a different context for the game."

–Brandenburger, Friedenberg, and Keisler (*Econometrica*, 2008)

Epistemic analysis generally depends on the type structure used–in which case, the outcome of the analysis is under-determined by the classical game model.

The Context-Free Case

There are type structures that, in one or another sense, contain all possible beliefs:

- **terminal** structures (Böge and Eisele, 1979);
- **canonically-built** (aka universal) structures (Mertens and Zamir, 1985);
- **complete** structures (Brandenburger, 2003).

Epistemic analysis on such structures can yield sharp results–see Battigalli and Siniscalchi (2002) and other papers.

Ellsberg on Uncertainty

> "These particular uncertainties–as to the other players' beliefs about oneself–are almost universal, and it would constrict the application of a game theory fatally to rule them out."

–Daniel Ellsberg (*The Review of Economics and Statistics*, 1959)

A player's **conjecture** is a probability measure on the strategy profiles chosen by the other players.

Fact (Aumann and Brandenburger, 1995)

Every (mixed-strategy) Nash equilibrium can arise in an epistemic structure where, at the true state, each player assigns probability 1 to the actual conjectures. (Moreover, each player assigns probability 1 to this event, and so on.)

So, Nash equilibrium does not (intrinsically) allow for the kind of uncertainty Ellsberg wanted.