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Adam Brandenburger

J.P. Valles Professor

Stern School of Business, New York University, USA

Why were you initially drawn to game theory?

I first saw game theory in a lecture given by Frank Hahn at Cambridge University, where I was an undergraduate. Hahn drew a 2×2 coordination game on the blackboard (see Figure 1) and explained that while the $(2, 2)$ outcome was the 'obvious' one in this game, the $(1, 1)$ outcome was also possible. If Ann believes that Bob will play R, she will optimally play D. If Bob believes that Ann will play D, he will optimally play R. The outcome of the game depends on what the players believe about the game, not just on the 'material' payoffs.

		Bob	
		L	R
Ann	U	$2, 2$	$0, 0$
	D	$0, 0$	$1, 1$

Figure 1

Much later, I found the wonderful passage in von Neumann and Morgenstern [18, 1944, p.42], which, I think, says the same thing (admittedly, in the context of cooperative rather than noncooperative game theory):

> [W]e shall in most cases observe a multiplicity of solutions. Considering what we have said about interpreting solutions as stable "standards of behavior" this has a simple and not unreasonable meaning, namely that given the same physical background different "established orders of society" or "accepted standards of behavior" can be built. . . .

Having come from a science background, I was intrigued by the idea of this dependence on the 'ethereal'—on the standards or beliefs to which people adhere. (In game theory, the dependence can even be on what people believe that other people believe, and so on.) Moreover, mathematics could be used to talk about this idea.

Of course, it has taken a long time for researchers to build a systematic game theory – nowadays called the epistemic approach to game theory – that incorporates these ideas. I have written two surveys – one in 1992 ("Knowledge and Equilibrium in Games" [4, 1992]) and one recently ("The Power of Paradox: Some Recent Developments in Interactive Epistemology" [5, to appear]) – which are my attempts to describe this intellectual journey.

What example(s) from your work (or the work of others) illustrates the use of game theory for foundational studies and/or applications?

Game theory is well suited to foundational work in a variety of fields. In my work, I have concentrated on the foundations of these foundations, so to speak. This work asks what sounds like a very classical question: What is rational behavior in a game—where each player thinks about the rationality of the other players, and so on? In fact, we still don't have a complete answer to this question.

In "Lexicographic Probabilities and Choice under Uncertainty" [3, 1991], Larry Blume, Eddie Dekel, and I developed an extension of probability theory, designed to tackle the following aspect of the rationality question. In the game of Figure 2, R is inadmissible (i.e., weakly dominated) for Bob. If Ann thinks that Bob adheres to an admissibility requirement, then, presumably, she should put probability 0 on R, and so will rationally play U. But admissibility seems to require that Ann put positive weight on both L and R. (This is because of the standard equivalence for finite games: A strategy s for Ann is admissible if and only if there is a full-support measure on Bob's strategy set under which s is optimal.) If Ann puts sufficient weight on R, she will play D, not U. This is the conceptual problem with admissibility. (See Samuelson [17, 1992]; also the discussion below of my paper [6, 2006] with Amanda Friedenberg and H. Jerome Keisler.)

		Bob	
		L	R
Ann	U	$2, 2$	$0, 0$
	D	$1, 1$	$1, 1$

Figure 2

Admissibility plays an important role in game theory. In applications, many games can be successfully analyzed this way. At the foundational level, admissibility is related to the very important idea of invariance (Kohlberg and Mertens [12, 1986, Section 2.4]). In [3, 1991], Blume, Dekel, and I proposed a solution to the admissibility problem that involves "lexicographic probability systems" (LPS's): Ann has a sequence of probability measures corresponding to a primary hypothesis about the game, an (infinitely less likely) secondary hypothesis, and so on. In Figure 2, Ann could have a primary hypothesis that puts weight 1 on L, and a secondary hypothesis that puts weight 1 on R. This way, Ann both excludes Bob's inadmissible strategy R (because it gets only infinitesimal weight) and includes it (because it nevertheless gets positive weight).

In terms of probability theory, LPS's are related to the extension of Kolmogorov theory developed by Renyí [16, 1955].

Apart from dominance ideas (such as admissibility), the other central concept in non-cooperative game theory is, of course, Nash equilibrium. In "Epistemic Conditions for Nash Equilibrium" [2, 1995], Robert Aumann and I provided conditions for this concept. The case of pure equilibrium is immediate: If each player is rational and assigns probability 1 to the actual strategy choices of the other players, then the strategies constitute a Nash equilibrium. (We gave an example above, with the strategies D and R in Figure 1.) Mixed equilibrium is more subtle. For this, we built on Harsanyi's proposal [10, 1973] to turn randomization around, so to speak, and treat it instead as uncertainty on the part of other players about a given player's definite choice of (pure) strategy. We provided two results on mixed equilibria—with different conditions for two-player games and for games with three or more players.

Recently, Friedenberg, Keisler, and I have returned to the problem of admissibility ("Admissibility in Games" [6, 2006]). The problem of iterated admissibility–i.e., the iterated removal of inadmissible strategies–has remained largely open. We provide epistemic conditions involving "rationality and mth-order assumption of rationality" (where "assumption" is a concept based on LPS's) and "completeness" (a kind of richness condition). We also uncover an impossibility result: Under a nontriviality condition, rationality and common assumption of rationality (mth-order assumption of rationality for all m) is impossible under completeness. We interpret this result as indicating a limit to the idea that players reason about all possibilities in a game. In this sense, rationality, even as a theoretical concept, appears to be inherently limited.

If trying to summarize the epistemic program to date, I would point to a progressive expansion in the concept of a game: The classical matrix or tree has been augmented by structures that enable us, the analysts, to talk about the players' rationality, knowledge, beliefs, assumptions, etc. In the classical treatment, these components weren't treated formally and were poorly understood. Borrowing from an essay by Gray [9, 2001, p.866] in the *American Mathematical Monthly*:

> There are intuitions and representations, and the representations may not capture the intuitions.

The idea that Ann thinks about Bob, and about what Bob thinks about her, and so on, has always been a basic intuition about games. With the epistemic program, it has become possible to represent these intuitions within the formal theory.

What is the proper role of game theory in relation to other disciplines?

In the predecessor volume to this one, Keisler [11, 2005, p.119] wrote in answer to the parallel question about the role of mathematics in general:

> I view mathematical research as exploring mathematical intuitions ... . Formal systems are used to clarify, sharpen, and communicate intuitive observations.

Game theory is often used in this fashion. In a pair of papers ("Value-Based Business Strategy" [7, 1996] and "Biform Games" [8, to appear]), Harborne Stuart and I have used game theory this way to explore some ideas in the area of business strategy. Here, the mathematical intuitions are about notions of "competition," "competitive position," and the like. We provide some formal structure to define and analyze these notions.

A particular feature of this work is that we develop and apply a hybrid noncooperative-cooperative formalism. The noncooperative moves describe the players' strategic moves. The consequence of these moves is a particular market structure, formalized as a cooperative game. (See below for more on the cooperative model–including its use in analyzing competition.)

John Geanakoplos (Yale) once posed the following puzzle to me: In the area of business strategy, people often talk about good strategy as "choosing the right game to play." But isn't such a choice just a move in a larger game? One answer is that the game being chosen is the cooperative game to be played, while the act of choosing the game to be played is a noncooperative move. This creates a formal distinction between playing and choosing a game, which also seems to fit well with usage in business strategy. In Keisler's terms, a formal system is used to clarify an intuitive idea.

Of course, there are many other examples in many different fields where game theory is used for a similar purpose.

What do you consider the most neglected topics and/or contributions in late 20th century game theory?

I think that cooperative theory in general is a neglected area.

Von Neumann and Morgenstern [18, 1944, p.529] defined a cooperative game starting from a noncooperative game. (The characteristic function for a subset A of players is the maximin payoff to A in the associated zero-sum game between A and not-A.) But later in their book [18, 1944, p.555 on], when they come to discuss market models such as bilateral monopoly and oligopoly, the characteristic function appears as a primitive. The image is of a 'free-form' market–without delineated moves for the players. Instead, the market is a fluid process where the value that each subset of players can create (i.e., the value of the characteristic function for each subset) determines the outcome.

The core is, perhaps, the most basic solution concept in this setting: Each subset must capture at least as much value as it creates. Aumann [1, 1985, p.53] explains that "the core expresses the idea of unbridled competition." It is striking how much insight can be got from using just the formalism of the characteristic function and the core inequalities. There are the famous core convergence and equivalence results, of course. But there are also subtle analyses of markets with small numbers–a fascinating example is Postlewaite and Rosenthal [14, 1974].

Still, I think it is correct to say that the contributions of cooperative theory have not been as widely known or taught as those of noncooperative theory. Happily, this might be changing, which I think would be a very positive development.

What are the most important open problems in game theory and what are the prospects for progress?

I will mention what I see as an open area in epistemic game theory.

One of the motivations for the epistemic program is empirical, or at least quasi-empirical. Considerations of the "I think you think..." kind seem very natural and basic in a game situation. Morgenstern [13, 1928, p.98] wrote about the battle of wits between Sherlock Holmes and Professor Moriarty (from *The Adventure of the Final Problem*) in exactly these terms.

An open area, then, as I see it, is making connections between epistemic game theory and empirical work, including experimental work. There is an experimental field – called Theory of Mind (Premack and Woodruff [15, 1978]) – which is very intriguing in this regard. This field examines the ability of humans (and nonhumans, such as chimpanzees) to recognize that others may have different "mental states" from one's own. An example is recognizing that others may not know something that one knows oneself.

Of course, this leads to questions such as: Are people – some people? – able even to think about other people's mental states about yet other people's mental states? And so on. Epistemic game theory is a formal language that expresses these possibilities and works out implications for strategic interactions. The opportunity, then, is to add empirical content to this language.

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