Game Theory and Business Strategy

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Origins of Game Theory

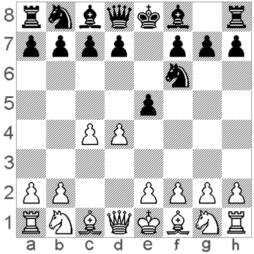
In the first decades of the twentieth century, chess enjoyed great visibility in many parts of Continental Europe.... From London to Moscow, the grand masters enjoyed great visibility and prestige, and the game was played in the chess cafés of the capitals, such as the Marienbrücke in Vienna and the famous Café de la Régence in Paris. Against a background of high tournament drama, chessmasters such as [Emanuel] Lasker and Siegbert Tarrasch wrote manuals on strategy, and the influence of the game was felt in many dimensions of scientific and literary culture. Thus psychologists investigated the thought processes required in chess, and mathematicians wondered whether so human an activity could be made amenable to formal treatment. Others speculated philosophically about the relationship of chess to life in general, and the game was a source of inspiration for several writers, including Vladimir Nabokov ... and ... Stefan Zweig.

–*Von Neumann, Morgenstern, and the Creation of Game Theory*, by Robert Leonard, Cambridge University Press, 2010, p.10

Chess and Life

> *The magnitude of the work that a group of [players] can perform under all varying possible conditions that may present themselves . . . is an index of the . . . value of that group.*

–*Struggle*, by Emanuel Lasker, Lasker's Publishing Company, New York, 1907, p.31 (Lasker was World Chess Champion from 1897 to 1921)



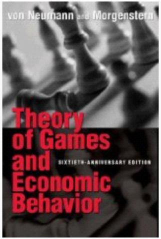
Source: Wikimedia Commons

Von Neumann's 1928 Paper

1. The concept of **strategy**
   "[I]t is possible to bring all games ... into a much simpler normal form .... Each player S_m $(m = 1, 2, \ldots, n)$ chooses a number $1, 2, \ldots, N_m$ without knowing the choices of the others."
2. The **Minimax Theorem**
   "[H]e is protected against his adversary 'finding him out.'"
3. The concept of a **cooperative** game
   "[T]he three-person game is essentially different from a game between two persons. ... It is [now] a question of which of the three equally possible coalitions $S_1, S_2; S_1, S_3; S_2, S_3$ has been formed. A new element enters, which is entirely foreign to the stereotyped and well-balanced two-person game: struggle."

Theory of Games and Economic Behavior

The theory of mechanics for $2, 3, 4, \ldots$ *bodies is well known, and in its general theoretical (as distinguished from its special and computational) form is the foundation of the statistical theory for great numbers. For the social exchange economy–i.e., for the equivalent 'games of strategy'–the theory of* $2, 3, 4, \ldots$ *participants was heretofore lacking.... A fundamental reopening of this subject is the more desirable because it is neither certain nor probable that a mere increase in the number of participants will always lead in fine to the conditions of free competition. (pp.14-15)*



Method of description:

(i) A set N of **players**;

(ii) A (**characteristic**) function $v : 2^N \to \mathbb{R}$ where, for each subset S, the number $v(S)$ is interpreted as the value created by the players in S.

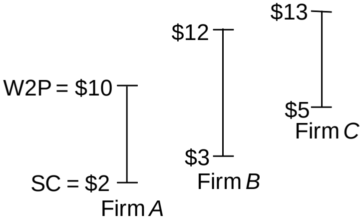
Methods of analysis:

(i) Core, Shapley Value, ...

(ii) *"The core expresses the idea of unbridled competition."**

* "What is Game Theory Trying to Accomplish?" by R. Aumann, in K. Arrow and S. Honkapohja, eds., *Frontiers of Economics*, Basil Blackwell, 1985, p.53

Example #1



3 firms, 2 buyers, many suppliers (all 'unitary')
Total value $v(N) = \$17$
Marginal contributions ("added values") $v(N) - v(N \setminus \{i\})$:

Firm A = Firm C = Each supplier = $0

Firm B = $1

Each buyer = $8

Hint: Apply the Goldilocks Principle from astronomy! (My thanks to Laura Needham)

The Linguistic Connection

Some frameworks from business strategy:

Five Forces (Porter 1980)

Imitation-Substitution-Holdup-Slack (Ghemawat 1991) . . .

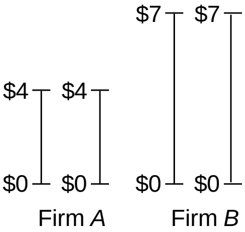
Language from business strategy:

Value, power, bargaining, negotiation, . . .

Language of cooperative game theory:

Very similar!

Example #2



2 firms (each with capacity 2) and 3 buyers
Total value $v(N) = \$18$
Firm A gets \$0, firm B gets \$6, each buyer gets \$4

Reference: "Disadvantageous Syndicates," by A. Postlewaite and R. Rosenthal, *Journal of Economic Theory*, 9, 1974, 324-326

The Conceptual Connection

> *[C]ooperative theory starts out with a formalization of games . . . that abstracts away altogether from procedures and ... concentrates, instead, on the possibilities for agreement.*

–"Game Theory," by R. Aumann, in J. Eatwell, M. Milgate, and P. Newman, *The New Palgrave: Game Theory*, Norton, 1989, p.8

> *[Non-cooperative] game-theoretic techniques require clear and distinct 'rules of the game.' Analysis of free-form competition . . . is not within the realm of the techniques provided.*

–*Game Theory and Economic Modelling*, by D. Kreps, Oxford University Press, 1990, pp.94-95

Choosing the Game

What about "strategic moves" such as the decision

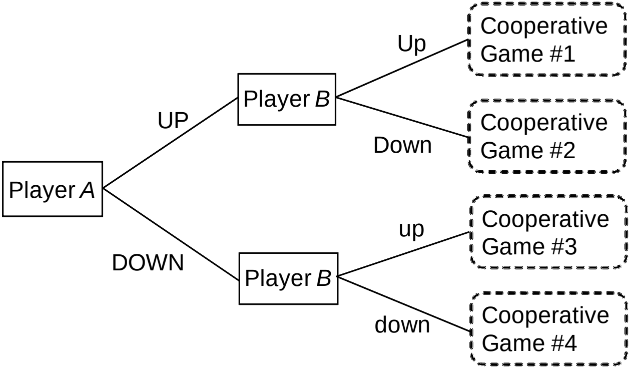
- whether to enter a market
- where to position a product
- what brand to build
- how much capacity to install
- how much money to devote to R&D ...

Such moves and countermoves are naturally formalized via non-cooperative theory

But we want to analyze the consequences via cooperative theory

So, we need a hybrid formalism

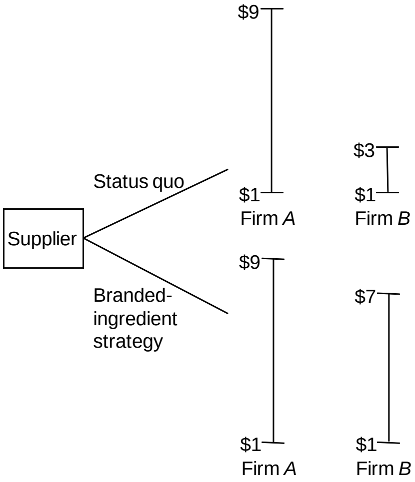
Biform Games*



Gluing together the different formalisms gives a meaning to strategy as "choosing the game"

* This and the next several slides are based on "Biform Games," by A. Brandenburger and H. Stuart, *Management Science*, 53, 2007, 537-549

Example #3*



1 supplier, 2 firms, many buyers (all 'unitary')

* From teaching materials by A. Brandenburger and K. Corts

Aside on Non-Cooperative Game Theory

The biform framework is not restricted to using

- Nash equilibrium
- Backward induction

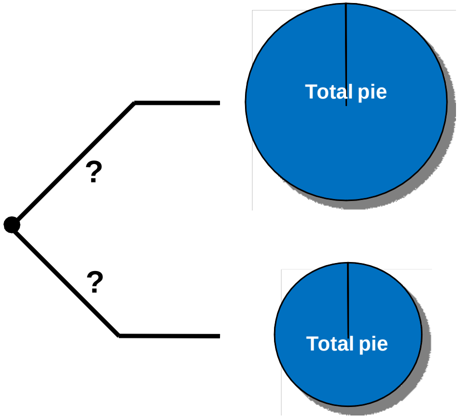
The **epistemic program** in game theory

(i) establishes that these solution concepts do not follow from the assumption that the players are rational (but from much more stringent assumptions)

(ii) develops a number of alternative solution concepts*

* For a survey, see "The Power of Paradox: Some Recent Results in Interactive Epistemology," by A. Brandenburger, *International Journal of Game Theory*, 35, 2007, 465-492

Efficiency and Inefficiency



A strategy profile $(s^1, \ldots, s^n)$ is **efficient** (resp. **inefficient**) if it leads to (resp. does not lead to) the largest total value

Conditions on Biform Games

Definition (Adding Up)

For each strategy profile s,
$\sum_{i=1}^{n}[V(s)(N) - V(s)(N\backslash\{i\})] = V(s)(N)$.

Definition (No Externalities)

For each player i, pair of strategies r^i, s^i for i, and strategy profile s^{-i} for the players other than i,
$V(r^i, s^{-i})(N\backslash\{i\}) = V(s^i, s^{-i})(N\backslash\{i\})$.

Definition (No Coordination)

For each player i, pair of strategies r^i, s^i for i, and pair of strategy profiles r^{-i}, s^{-i} for the players other than i,
$V(r^i, r^{-i})(N) > V(s^i, r^{-i})(N)$ iff $V(r^i, s^{-i})(N) > V(s^i, s^{-i})(N)$.

Game-Theoretic Welfare Theorems

Theorem

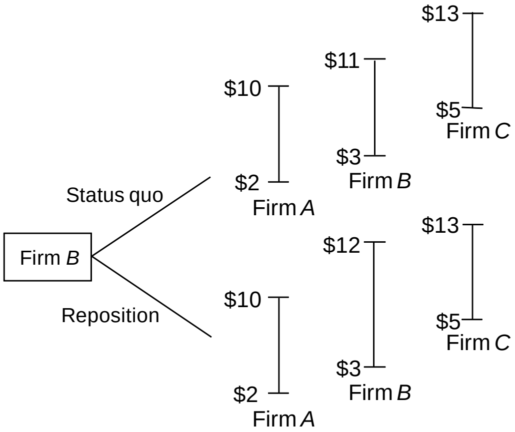
Fix a biform game satisfying AU, NE, and NC (and non-emptiness of the Core for each strategy profile). Then, if a profile s is a (pure) Nash equilibrium, it is efficient.

Theorem

Fix a biform game satisfying AU and NE (and non-emptiness of the Core for each strategy profile). Then, if a profile s is efficient, it is a (pure) Nash equilibrium.

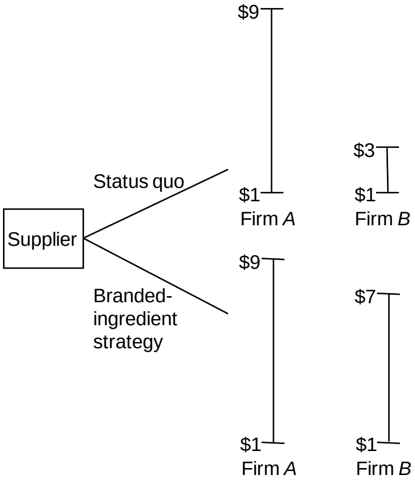
Note: These theorems are closely related to results in "Appropriation and Efficiency: Revision of the First Theorem of Welfare Economics," by L. Makowski and J. Ostroy, *American Economic Review*, 85, 1995, 808-827

Example #1 cont'd



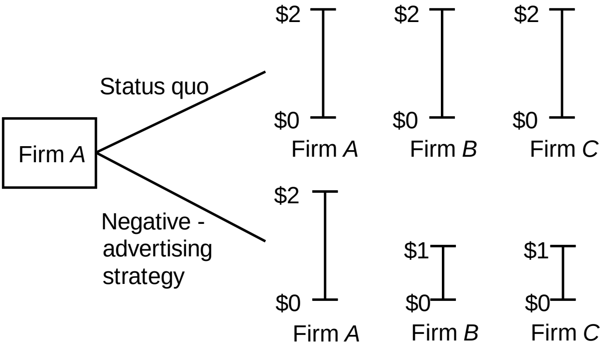
This game satisfies AU, NE, and NC

Example #3 cont'd



This game fails AU

Example #4



3 firms, 2 buyers, many suppliers (all 'unitary')
This game satisfies AU and NC, but fails NE

A Connection to Multibusiness Strategy

Think of the multibusiness firm as a "mini-economy" that may exhibit inefficiencies

Our theorem gives us a classification:

(i) Example of a failure of AU:
– The Holdup Problem leading to a failure to invest

(ii) Example of a failure of NE:
– Divisional vs. corporate management of brand, knowledge, . . . leading to a failure to account for spillovers

(iii) Example of a failure of NC:
– Divisional vs. "coordinated" choice of supplier leading to a failure to defray a supplier's fixed costs, speed up its learning, . . .

Appendix: Evaluation of Cooperative Games

The bargaining process in a cooperative game can be decomposed into two elements:

(i) Competition–captured via the Core

(ii) Residual negotiation–if the Core is not a single point

A theory of residual negotiation:

a. For each player, calculate the Core projection (a closed bounded interval of $\mathbb{R}$)
b. Take a (subjective) weighted average of the upper and lower endpoints

This measure can be axiomatized via a modification of the Milnor (1954) derivation of the Hurwicz (1951) optimism-pessimism index

Appendix: When is Rationality Equivalent to Efficiency?

Definition

In a non-cooperative game, a player is **rational** if he chooses a strategy that maximizes his expected payoff, under some probability measure on the product of the strategy sets for the other players.

Theorem

Assume AU, NE and NC (and non-emptiness of the Core for each strategy profile). Then, a profile consists of rational strategies if and only if the profile is efficient.

Appendix: The Five Forces as Size and Division of the Pie

Threat of Substitutes
– affects W2P

Rivalry among Existing Competitors
– asks to what extent incumbent firms compete away value downstream or upstream

Bargaining Power of Buyers
– asks to what extent buyers can retain the value created for themselves

Threat of New Entrants
– asks to what extent new firms can enter and compete away value to buyers (bidding down prices) or to suppliers (bidding up costs)

Bargaining Power of Suppliers
– asks to what extent suppliers can retain the value created for themselves

Appendix: Assessment of the Five Forces

The Supplier Side?

– SC as well as W2P determines the size of the pie. Likewise, notice, the asymmetry in the names of the "Generic Strategies".

Decomposability of "power"?

– "Buyer Power" must depend on how many firms there are—and on their capacities etc. Similarly with "Supplier Power" and "Rivalry".

Other players?

– Complementors with respect to buyers—and with respect to suppliers—can matter, too.