Epistemic Game Theory

Adam Brandenburger

J.P. Valles Professor, NYU Stern School of Business
Distinguished Professor, NYU Tandon School of Engineering
Faculty Director, NYU Shanghai Program on Creativity + Innovation
Global Network Professor
New York University

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Von Neumann (1928)

n players $S_1, S_2, \ldots, S_n$ are playing a given game of strategy, $\mathcal{G}$. How must one of the participants, S_m, play in order to achieve a most advantageous result? ([26, 1928, p.13])

[I]t is inherent in the concept of "strategy" that all information about the actions of the participants and the outcomes of "draws" [i.e., moves by Nature] a player is able to obtain or infer is already incorporated in the "strategy." Consequently, each player must choose his strategy in complete ignorance of the choices of the rest of the players and of the results of the "draws." ([26, 1928, p.19])

In two-person zero-sum games, minimax theory avoids a predictive approach

In n-person nonzero-sum games, the focus is on combination and group minimax

Von Neumann, J., "Zur Theorie der Gesellschaftsspiele," *Mathematische Annalen*, 100, 1928, 295-320; Bargman, S., English translation: "On the Theory of Games of Strategy," in Tucker, A., and R.D. Luce (eds.), *Contributions to the Theory of Games*, Volume IV, Princeton University Press, 1955. 13-42; Von Neumann, J., and O. Morgenstern, *Theory of Games and Economic Behavior*, Princeton University Press, 1944

Nash (1951)

> We proceed by investigating the question: what would be a "rational" prediction of the behavior to be expected of rational playing the game in question? By using the principles that a rational prediction should be unique, that the players should be able to deduce and make use of it, and that such knowledge on the part of each player of what to expect the others to do should not lead him to act out of conformity with the prediction, one is led to the concept of a solution defined before.

In equilibrium theory, players are assumed to have access to the actual strategies chosen by the other players

Nash, J., "Non-Cooperative Games," doctoral dissertation, Princeton University, 1950; available at http://rbsc.princeton.edu/topics/nash-john-1928-2015; Nash, J., "Non-Cooperative Games," *Annals of Mathematics*, 54, 1951, 286-295

The Trilogy of Decision Theory

What if, instead, we equip players with probability measures over the strategy choices of the other players?

This leads naturally to the idea of a probability measure over a space of probability measures for the other players … and so on (Mertens and Zamir, 1985; Brandenburger and Dekel, 1993)

Epistemic game theory equips each player with

a strategy set (we will assume finite)

a payoff function

a hierarchy of beliefs over the strategies chosen

We can think of this as the multi-person analog to the “trilogy” of decision theory

Mertens, J.F., and S. Zamir, S. “Formulation of Bayesian Analysis for Games with Incomplete Information,” *International Journal of Game Theory*, 14, 1985, 1-29; Brandenburger, A., and E. Dekel, “Hierarchies of Beliefs and Common Knowledge,” *Journal of Economic Theory*, 59, 1993, 189-198

An Epistemic Game

	L	R
U	2, 2	0, 0
D	0, 0	1, 1

$\lambda^a(t^a)$

T^b \ S^b	L	R
u^b	0	1/2
t^b	0	1/2

$\lambda^a(u^a)$

T^b \ S^b	L	R
u^b	1/2	0
t^b	0	1/2

$\lambda^b(t^b)$

T^a \ S^a	U	D
u^a	0	1/2
t^a	0	1/2

$\lambda^b(u^b)$

T^a \ S^a	U	D
u^a	1/2	0
t^a	0	1/2

Epistemic type spaces T^a, T^b

with associated maps $\lambda^a : T^a \to \mathscr{M}(S^b \times T^b)$

$\lambda^b : T^b \to \mathscr{M}(S^a \times T^a)$

An Epistemic Game contd.

	L	R
U	2, 2	0, 0
D	0, 0	1, 1

$\lambda^a(t^a)$

T^b \ S^b	L	R
u^b	0	1/2
t^b	0	1/2

$\lambda^a(u^a)$

T^b \ S^b	L	R
u^b	1/2	0
t^b	0	1/2

$\lambda^b(t^b)$

T^a \ S^a	U	D
u^a	0	1/2
t^a	0	1/2

$\lambda^b(u^b)$

T^a \ S^a	U	D
u^a	1/2	0
t^a	0	1/2

At the state (D, t^a, R, t^b)

Alice is ‘correct’ about Bob’s strategy

Bob is correct about Alice’s strategy

Alice, though, thinks it possible Bob is wrong about her strategy

Alice is rational

Bob is rational

Alice, though, thinks it possible Bob is irrational

Some Results in Epistemic Game Theory

The Fundamental Theorem of EGT: Under rationality and common belief of rationality (RCBR), each player will choose an iteratively undominated (IU) strategy. Conversely, given an IU strategy for each player, we can build a type structure so that these strategies are consistent with RCBR in the structure.

(Informal treatment — Bernheim, 1984; Pearce, 1984

Formal treatment — Brandenburger and Dekel, 1987; Tan and Werlang, 1988)

Conditions for Nash equilibrium: Aumann and Brandenburger (1995)

Conditions for extensive-form rationalizability: Battigalli and Siniscalchi (2002)

Condition for iterated admissibility: Brandenburger, Friedenberg, and Keisler (2008)

Bernheim, D., "Rationalizable Strategic Behavior," *Econometrica*, 52, 1984, 1007-1028; Pearce, D., "Rationalizable Strategic Behavior and the Problem of Perfection," *Econometrica*, 52, 1984, 1029-1050; Brandenburger, A., and E. Dekel, "Rationalizability and Correlated Equilibria," *Econometrica*, 55, 1987, 1391-1402; Tan, T., and S. Werlang, "The Bayesian Foundations of Solution Concepts of Games," *Journal of Economic Theory*, 45, 1988, 370-391; Aumann, R., and A. Brandenburger, "Epistemic Conditions for Nash Equilibrium," *Econometrica*, 63, 1995, 1161-1180; Battigalli, P., and M. Siniscalchi, "Strong Belief and Forward Induction Reasoning," *Journal of Economic Theory*, 2002, 106, 356-391; Brandenburger, A., A. Friedenberg, and H.J. Keisler, Admissibility in Games," *Econometrica*, 76, 2008, 307-352.

Beyond Conventional Game Theory: Correlation in EGT

Correlation is basic to non-cooperative game theory

For example, consider the equivalence between not-strongly dominated strategies and strategies that are optimal under some probability measure on the strategy profiles of the other players

As is well known, for this equivalence to hold in games with more than two players, the probability measure may need to be dependent (i.e., correlated)

Brandenburger, A., and A. Friedenberg, "Intrinsic Correlation in Games," *Journal of Economic Theory*, 141, 2008, 28-67

Example of Correlation

	L	R
U	1,1,3	1,0,3
D	0,1,0	0,0,0

X

	L	R
U	1,1,2	0,0,0
D	0,0,0	1,1,2

Y

	L	R
U	1,1,0	1,0,0
D	0,1,3	0,0,3

Z

Alice's strategy set $S^a = \{U, D\}$

Bob's strategy set $S^b = \{L, R\}$

Charlie's strategy set $S^c = \{X, Y, Z\}$

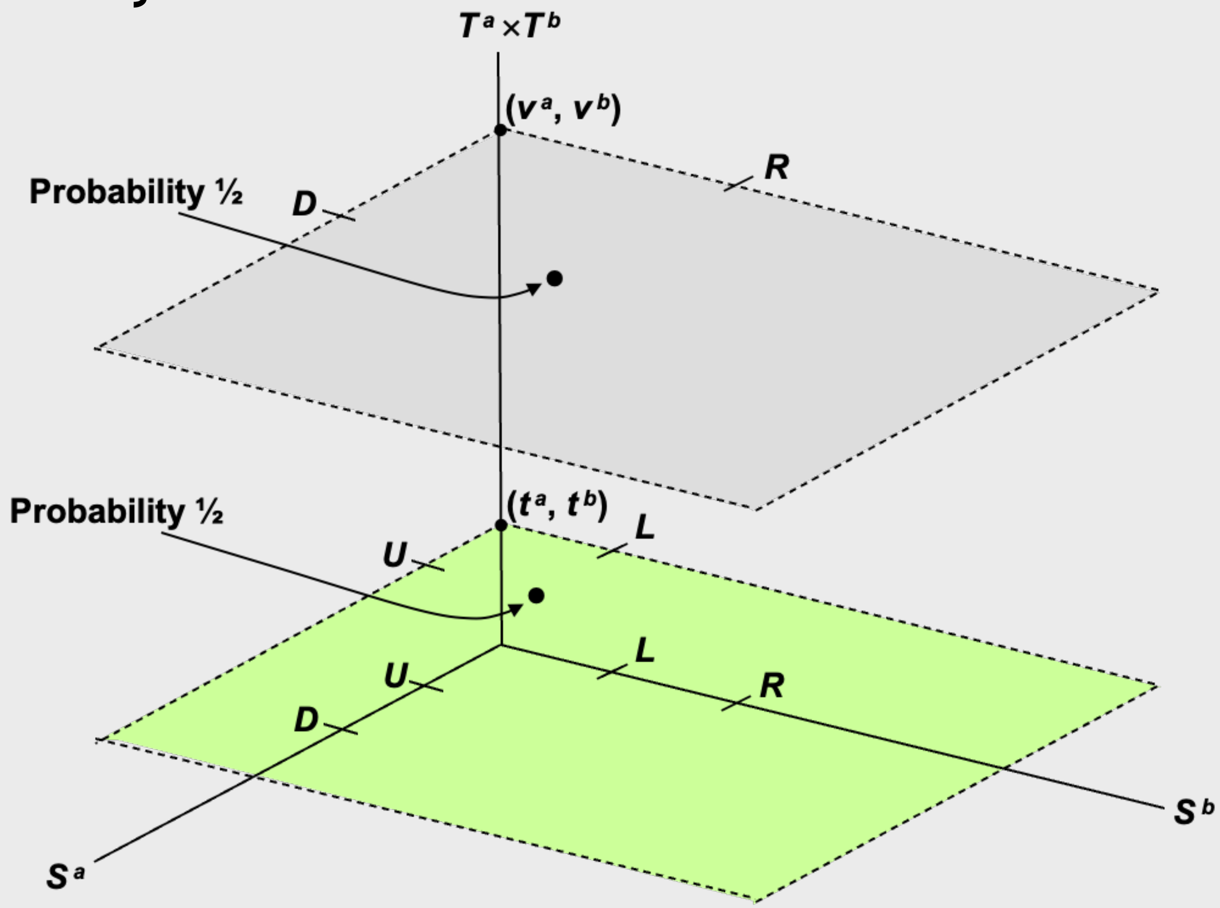
Alice's type set $T^a = \{t^a, v^a\}$

Bob's type set $T^b = \{t^b, v^b\}$

Charlie's type set $T^c = \{t^c\}$

Brandenburger and Friedenberg op.cit.

Type Space for Player c



Type t^c assigns probability $1/2$ to (U, t^a, L, t^b)
and probability $1/2$ to (D, v^a, R, v^b)

Conditions on Type Structures

Conditional Independence (CI):

Charlie's type t^c should satisfy

$$p(s^a, s^b \,|\, t^a, t^b) = p(s^a \,|\, t^a, t^b) \times p(s^b \,|\, t^a, t^b) \text{ whenever } p(t^a, t^b) > 0$$

(likewise for Ann and Bob)

Sufficiency (SUFF):

Charlie's type t^c should satisfy

$$p(s^a \,|\, t^a, t^b) = p(s^a \,|\, t^a) \text{ whenever } p(t^a, t^b) > 0$$

(likewise for b, and for Ann and Bob)

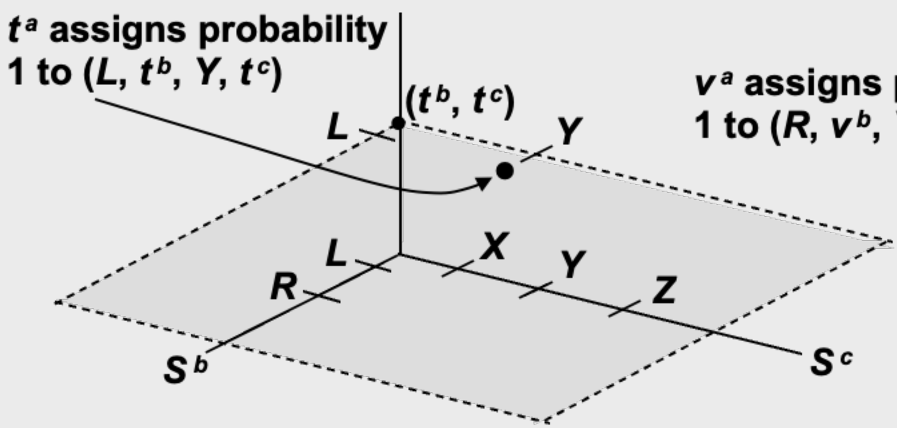
Under CI and SUFF:

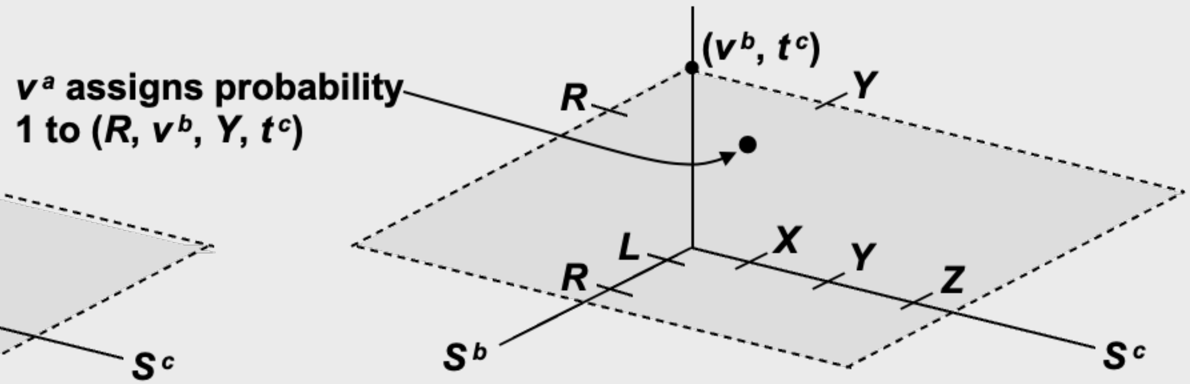
If $p(t^a, t^b) = p(t^a) \times p(t^b)$, then $p(s^a, s^b) = p(s^a) \times p(s^b)$

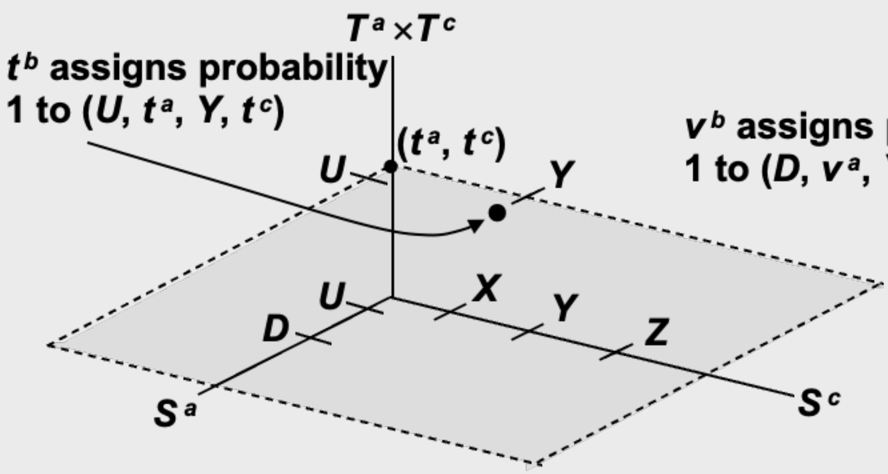
In words, a correlated assessment about strategies implies a correlated assessment about types (no physical correlation)

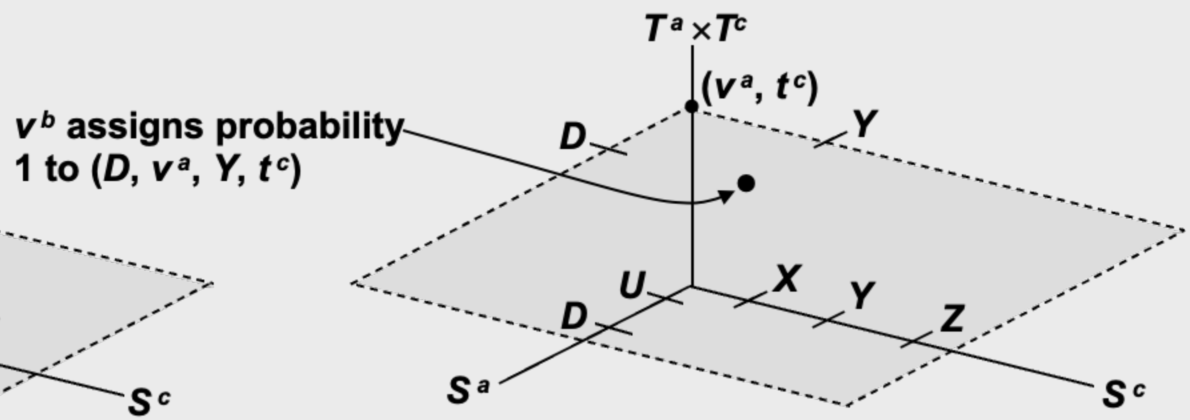
Brandenburger and Friedenberg op.cit.

Type Spaces for Players *a* and *b*









Brandenburger and Friedenberg op.cit.

Back to the Example of Correlation

All types of each player satisfy CI and SUFF

For Charlie, the strategy-type pair (Y, t^c) is rational

For Alice, the strategy-type pairs (U, t^a) and (D, u^a) are rational

For Bob, the strategy-type pairs (L, t^b) and (R, u^b) are rational

Also, each type for each player assigns positive probability only to rational strategy-type pairs for the other players

By induction, each of these strategy-type pairs is therefore consistent with RCBR

In particular, Charlie can play Y under RCBR

Technical notes:

(i) The formal treatment of CI and SUFF involves bimeasurability of the type maps λ^i

(ii) If there are redundant types, then the conditioning must be on hierarchies

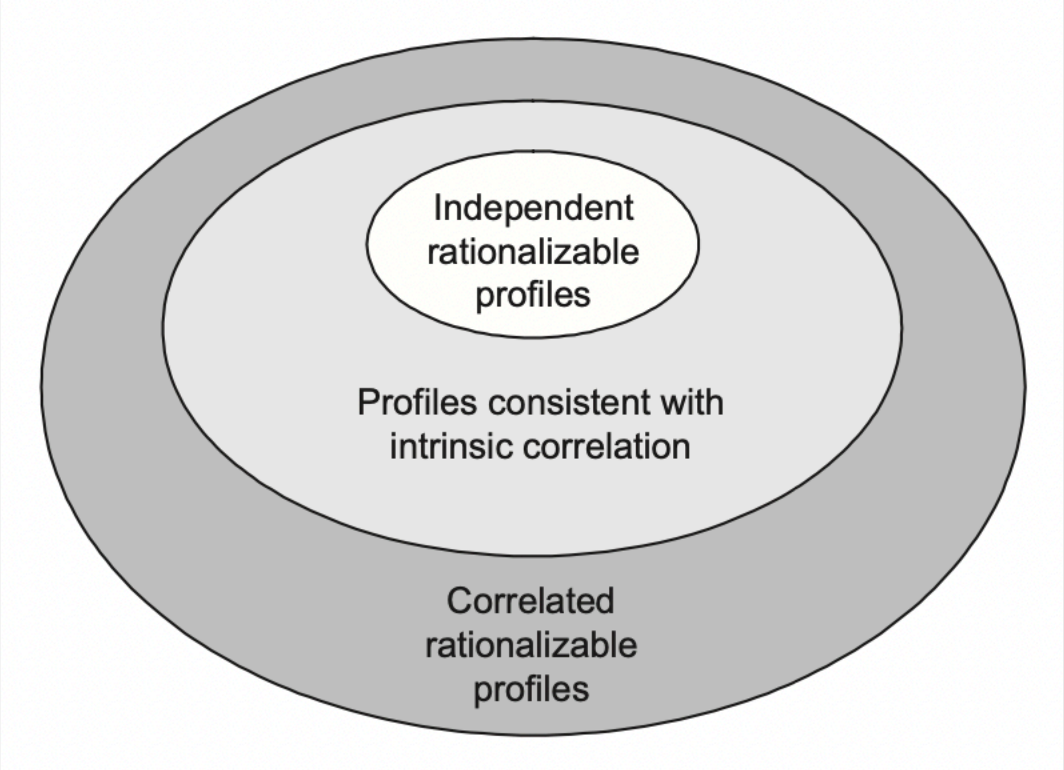
(iii) The full definitions of CI and SUFF are on infinite type spaces

Brandenburger and Friedenberg op.cit.

The Question

What strategies can be played in a game under the requirements of CI, SUFF, and RCBR — call this "intrinsic correlation"?

What we know so far:



Note: Brandenburger and Dekel (1987) show that correlated rationalizability (= IU) is equivalent to subjective correlated equilibrium (Aumann, 1974)

Brandenburger and Friedenberg op.cit.; Aumann, R., "Subjectivity and Correlation in Randomized Strategies," *Journal of Mathematical Economics*, 1, 1974, 76-96

Second Example of Correlation

	L	C	R
U	0,0,2	0,0,2	0,1,2
M	0,0,0	0,0,0	0,1,0
D	1,0,2	1,0,2	1,1,2

X

	L	C	R
U	1,1,1	0,1,0	0,1,0
M	1,0,0	0,0,1	0,1,0
D	1,0,0	1,0,0	1,1,0

Y

	L	C	R
U	0,0,0	0,0,0	0,1,0
M	0,0,2	0,0,2	0,1,2
D	1,0,2	1,0,2	1,1,2

Z

U and M are each optimal if and only if $\mathrm{Prob}(L, Y) = 1$

L and C are each optimal if and only if $\mathrm{Prob}(U, Y) = 1$

Y is optimal if and only if $\mathrm{Prob}(U, L) = \mathrm{Prob}(M, C) = 1/2$

Every strategy is iteratively undominated

Brandenburger and Friedenberg op.cit.

An Impossibility Theorem

Informally:

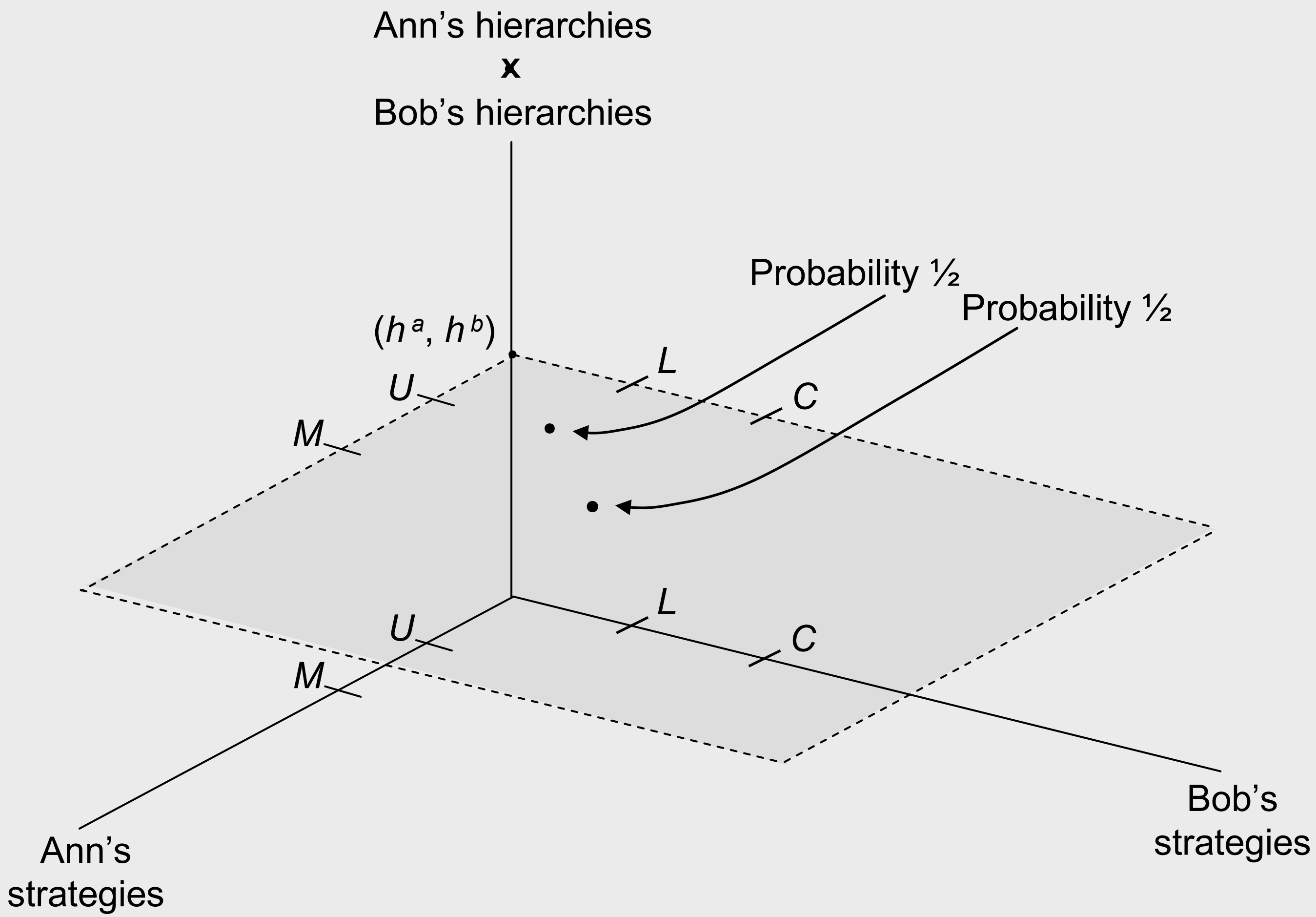
If (U, t^a) and (M, v^a) satisfy RCBR (for Alice), then the associated hierarchies of beliefs are equal

If (L, t^b) and (C, v^b) satisfy RCBR (for Bob), then the associated hierarchies of beliefs are equal

Therefore, if (Y, t^c) satisfies RCBR (for Charlie), the picture must be as follows …

Brandenburger and Friedenberg op.cit.

An Impossibility Theorem contd.



But CI requires Charlie's conditional measure, conditional on any horizontal plane, to be a product measure — contradiction!

Brandenburger and Friedenberg op.cit.

An Impossibility Theorem contd.

Formally:

There is a game G and a correlated rationalizable (= IU) strategy s^i of G, such that the following holds: For any type structure, there does not exist a state at which each type satisfies CI, RCBR holds, and s^i is played

There is an analogous theorem for SUFF

Notes:

The implication is that our theory of intrinsic correlation is distinct from the existing theory of correlation in non-cooperative games

Epistemic game theory is central to discovering this new theory

Brandenburger and Friedenberg op.cit.

Routes to Correlation

Independent rationalizability (no correlation!)

Intrinsic correlation (our route)

Correlated rationalizability (extrinsic correlation à la Aumann signals)

Brandenburger and Friedenberg op.cit.

Routes to Correlation contd.

Q: What are the implications of intrinsic correlation in economic settings — including macroeconomic settings?

(Example? Charlie thinks Alice and Bob either both do not change prices or both change prices, because he thinks their hierarchies of beliefs about other price movements are correlated)

A fourth route: Physical correlation

Charlie thinks that Alice and Bob jointly choose their strategies, even though he makes his decision in his own "cubicle"

This asymmetry can be avoided by moving to cooperative game theory, in which joint action by all subsets of players is considered

At the least, the boundary between non-cooperative and cooperative game theory is more blurred than usually thought

Q: Could cooperative game theory find new applications in macroeconomics?