Online Appendix
Coordination via Delay: Theory and Experiment

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1 Theory

1.1 A General Model

Here, we consider a general binary-action coordination game. Under the assumption of ϵ-social preference and binary message $m = 0, 1$, we will show that our main result holds true in this class of coordination game.

There are N players, indexed by $i \in \mathcal{N} := \{1, 2, \ldots, N\}$. Player i's final choice between A and B is denoted by d_i. The monetary payoff from the choice $d_i = B$ is fixed at b regardless of other players' choices d_{-i} – i.e., $\pi_i(d_i = B, d_{-i}) = b$ for any d_{-i}. On the other hand, the monetary payoff from $d_i = A$ is $\pi_i(d_i = A, d_{-i}) = a_k$, where $k \in \{1, , 2, \ldots, N\}$ denotes the total number of A choices in the action profile $d = (d_i, d_{-i})$; that is, $k = \big|\{j \in \mathcal{N} | d_j = A\}\big|$.

Strategic complementarity implies that $a_1 \leq a_2 \leq \ldots \leq a_N$. Some of the inequalities must hold strictly as $a_1 < b < a_N$ always holds for this coordination game.[1] Therefore, under this assumption, when players move simultaneously, all players taking A and all of them taking B are both pure-strategy Nash Equilibrium.

Based on this payoff structure, there always exists a unique $k_0 \in \{1, 2, \ldots, N-1\}$ such that $a_{k_0} \leq b < a_{k_0+1}$. Intuitively, player i would take A if knowing that at least k_0 (among $N-1$) other players will do the same. It is worth noting that our benchmark model is a special case where $k_0 = N-1$ and $a_1 = a_2 = \ldots = a_{N-1} = c < b < a_N = a$.

As in our benchmark model, each player can postpone their choices between A and B to $t = 1$ by choosing "wait" at $t = 0$. The message they can observe after waiting is binary, which depends on the number of B choices at $t = 0$, $n(s_{-i})$. Formally, for any player i who chooses to wait,

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[1] Otherwise, A is the (weakly) dominant action if $b \leq a_1$, and B is (weakly) dominant if $a_N \leq b$.

$m \equiv \mathbb{1}\{n(s_{-i}) \geq N - k_0\}$. Intuitively, if $m = 1$, then the total number of A choices, regardless of the actions taken by the players who choose to wait at $t = 1$, could be at most k_0. That means, following the massage $m = 1$, B is a (weakly) preferred choice because $a_k \leq b$ for all $k \leq k_0$ and $a_k < b$ for some $k < k_0$.[2]

As before, we can denote all possible strategies in this extensive form game as $\mathcal{S} \equiv \{B, WBB, WBA, WAB, WAA\}$. Below, we show that our main result of Theorem 1 and the way of proving it can be extended to this binary-action coordination game.

Proposition 1 *The unique strategy profile that survives iterated weak dominance is* $(s_i = WBA)_{i=1}^N$. *Under this strategy profile, each player chooses* $d_i = A$ *and receives the highest possible monetary payoff* a_N.

Proof.

First round of elimination Consider any player i and any strategy profile played by other players $s_{-i} = (s_j)_{j \in \mathcal{N} \setminus \{i\}}$. We want to show that WAB (WAA) is weakly dominated by WBB (WBA). Consider two mutually exclusive and collectively exhaustive cases.

In the first case, $m = 1$; that is, the other players adopt the strategy profile $s_{-i} = (s_j)_{j \in \mathcal{N} \setminus \{i\}}$ that satisfies $n(s_{-i}) \equiv \big|\{j \in \mathcal{N} \setminus \{i\} | s_j = B\}\big| \geq N - k_0$. In this case, $\pi_i(s_i = WBB, s_{-i}) = b$, while $\pi_i(s_i = WAB, s_{-i}) = a_k$ for some $k \leq k_0$. We first consider the cases in which $k \leq k_0$ satisfies $\pi_i(s_i = WBB, s_{-i}) = b = \pi_i(s_i = WAB, s_{-i}) = a_k$. In these cases, any player $j \neq i$ will get the b no matter player i chooses WAB or WBB. Therefore, for any such strategy profile s_{-i}, player i will be indifferent between WAB and WBB. However, there always exists $k \leq k_0$ such that $a_k < b$. Clearly, $\pi_i(s_i = WBB, s_{-i}) > \pi_i(s_i = WAB, s_{-i})$ for any s_{-i} that lets $n(s_{-i})$ equal such a k. Based on the utility function u_i defined in (1), WBB weakly dominates WAB in this case.

In the other case, $m = 0$; that is, $s_{-i} = (s_j)_{j \in \mathcal{N} \setminus \{i\}}$ that satisfies $n(s_{-i}) < N - k_0$. Therefore, $\pi_i(s_i = WBB, s_{-i}) = \pi_i(s_i = WAB, s_{-i}) = b$ and $\pi_j(s_i = WBB, s_{-i}) = \pi_j(s_i = WAB, s_{-i})$ for any s_j and any $j \in \mathcal{N} \setminus \{i\}$.

Hence, WAB is weakly dominated by WBB. The same argument can be applied to show that WAA is weakly dominated by WBA. Following the exact same procedures in the proof of Theorem 1, we can show that strategy B, WBA and WBB cannot be eliminated at this round.

Second round of elimination The remaining strategies are B, WBB and WBA. We first compare strategy WBB to strategy B. Note that, for player i and any s_{-i}, $\pi_i(s_i = WBB, s_{-i}) = \pi_i(s_i = B, s_{-i}) = b$. We will look into how the other players' payoffs depend on player i's choice between B and WBB by considering three mutually exclusive and collectively exhaustive cases. First, observe that, for any s_{-i} that admits $n(s_{-i}) \geq N - k_0$ (or $n(s_{-i}) < N - k_0 - 1$), then $m = 1$

[2]Note that, based on the assumption $a_1 < b$, in the special case where $k_0 = 1$, $a_{k_0} < b$.

(or $m = 0$) regardless of player i choice between B and WBB. Therefore, other players' choices as well as their payoffs are unaffected by player i's choice between B and WBB.

Next, consider any s_{-i} that satisfies (1) $n(s_{-i}) = N - k_0 - 1$ and (2) no one chooses WBA (i.e., $\left|\{j \in \mathcal{N} \setminus \{i\} | s_j = WBA\}\right| = 0$). In this case, $s_i = B$ leads to $m = 1$ while $s_i = WBB$ leads to $m = 0$. Nevertheless, no one's payoff is affected by i's choice between WBB and B as all players will ultimately take action B regardless of m.

In the last case, consider s_{-i} that satisfies (1) $n(s_{-i}) = N - k_0 - 1$ and (2) there is at least one player chooses WBA. For this group of players (denoted by j_0, generically), if $s_i = B$ and then $m = 1$, they will choose B at $t = 1$, and, accordingly, the payoff would be $\pi_{j_0} = b$; otherwise, if $s_i = WBB$ and $m = 0$, then their choice at $t = 1$ would be A and the payoff would be $\pi_{j_0} = a_k$ where $k \leq k_0$. This is because the number of B choices at $t = 0$ is $N - k_0 - 1$ and at least one B choice at $t = 1$ from player i who takes WBB. By definition of k_0, we know that $a_k \leq b$ for any $k \leq k_0$ and the inequality holds strictly with some $k < k_0$. Therefore, under the assumption of ϵ-social preferences, WBB is weakly dominated by B in this step. Following the exact same procedures in the proof of Theorem 1, we can show that strategy B and WBA cannot be eliminated at this round.

Third round of elimination The remaining strategies are WBA and B. To see that B is weakly dominated by WBA, consider the following three mutually exclusive and collectively exhaustive cases. First, consider any s_{-i} admits $n(s_{-i}) \geq N - k_0$. In those cases, $m = 1$ and all players will eventually take B and receives the payoff b, regardless of player i's choice between B and WBA.

Second, consider the s_{-i} that satisfies $n(s_{-i}) = N - k_0 - 1$. If player i chooses B, then $m = 1$, and, accordingly, $\pi_i = \pi_{j_0} = b$ (player j_0 is a generic player who chooses WBA); otherwise, if $s_i = WBA$, then $m = 0$, and, therefore, $\pi_i = \pi_{j_0} = a_{k_0+1} > b$. (Note that the total number of B choices in this case is $N - k_0 - 1$, and, thus, the total number of A choices (form the WBA choosers) is $k_0 + 1$.) Therefore, WBA is preferred in this case.

In the last case, s_{-i} satisfies that $n(s_{-i}) < N - k_0 - 1$. Therefore, $m = 0$ regardless of player i's choice between B and WBA. It is easy to check that $\pi_i(s_i = B, s_{-i}) = b < \pi_i(s_i = WBA, s_{-i}) = a_{N-n(s_{-i})}$ and $\pi_{j_0}(s_i = B, s_{-i}) = a_{N-n(s_{-i})-1} \leq \pi_{j_0}(s_i = WBA, s_{-i}) = a_{N-n(s_{-i})}$. Therefore, WBA is preferred in this case as well. In total, B is weakly dominated by WBA.

Coordination outcome As we have just shown, the strategy WBA is the unique iteratedly undominated strategy. Under the strategy profile $(s_i = WBA)_{i \in \mathcal{N}}$, the realized choice for each player is A and, accordingly, $\pi_i = a_N$ for all $i \in \mathcal{N}$. ∎

Therefore, in this class of coordination game, the delay option can work to ensure an efficient outcome. To see that the binary information setting is crucial for this result, we consider a finer information setting below and present a counterexample to show that the delay option cannot guarantee the efficient outcome when finer information is available.

Counterexample under finer information Consider a game with $N = 3$ players and payoff parameters $a_1 < b < a_2 < a_3$. Let $WX_0X_1X_2$ denote the strategies involving waiting, in which $X_n = A, B$ is the action after observing n B choices in the first period. We are about to show that there exists iteratedly undominated strategy profiles that do not lead to Pareto-outcome.

First, note that any strategy involves waiting and $X_2 = A$ is weakly dominated in the first round of elimination by the ones that involves waiting and takes the same choices on X_0 and X_1, but chooses B after $n = 2$. Therefore, the remaining strategies are B, $WAAB$, $WABB$, $WBBB$, $WBAB$. Next, we show that none of the five strategies can be eliminated in the first round of elimination.

To see why these strategies cannot be eliminated, we will examine them one by one.

1. $WBAB$ and $WBAA$ are the best responses when one opponent plays $WBAB$ and the other plays a mixture of B and $WBAB$. However, $WBAB$ weakly dominates $WBAA$.

2. $WAAB$ and $WAAA$ are the best responses to a mixture of B and $WAAB$ and $WAAB$. However, $WAAB$ weakly dominates $WAAA$.

3. $WABB$ and $WABA$ are the best responses to a mixture of B and $WAAB$ and $WABB$. But $WABB$ weakly dominates $WABA$.

4. $WBBB$ is the unique best response to $WBBB$ amd a mixture of $WBBB$, $WBAB$ and B.

5. B cannot be eliminated either. Consider the case in which the opponents are using $WBBB$. In that case, $WAAB$ and $WABB$ generate payoff a_1, while B, $WBBB$, and $WBAB$ generate payoff $b > a_1$. Next, consider the case in which one opponent plays $WBBB$ and the other one takes B. In that case, $WAAB$ and $WBAB$ generate payoff a_1, while B, $WBBB$, and $WABB$ generate payoff $b > a_1$. Obviously, any strategy involving taking waiting and taking A after $n = 2$ cannot dominates B. (To see this, consider the case where both opponents take the strategy B.) Therefore, any strategy weakly dominates B puts 0 weight on $WAAB$, $WBAB$, and $WABB$. Therefore, the only remaining candidate strategy which can dominates B is $WBBB$.

6. To see that B is not dominated by WBB, consider the case in which both of the other two players choose $WABB$, choosing B and $WBBB$ generate the same monetary payoff for the player themselves, but $WBBB$ generate lower payoff for the two other players. Therefore, under the assumption of ϵ-social preference, B cannot be dominated by $WBBB$.

Therefore, each of the strategies B, $WBBB$, $WAAB$, $WABB$, $WBBB$ and $WBAB$ survives the first round of elimination. As discussed above, each strategy cannot be eliminated given other players play a mixed strategy consisting of some of these five strategies. Therefore, no further elimination can happen in the later rounds. That said, each players taking B, or $WBBB$, or $WBAB$ are all undominated strategy profiles, and any of these strategy profile leads to an inefficient outcome.

1.2 Alternative Reversibility Structure: Both Irreversible

Here, we consider a different irreversibility structure in which players can choose among A, B and "wait" at $t = 0$ and both A and B are binding choices. We will explore the cases both of binary message and of finer information. In the binary message case, players who choose to wait at $t = 0$ can only observe whether any other player has chosen B at $t = 0$ ($m = 1$) or not ($m = 0$). In the finer information case, they can observe the number of A, B and wait choices at $t = 0$.

Binary Message

Although players can choose A at $t = 0$, those choices is similar to the choices of "wait" with regard to generating the binary message m. It is easy to check that pledging to A earlier at $t = 0$ is weakly dominated (in the first round of elimination) by the strategy of waiting and then choosing B after $m = 1$ and choosing A after $m = 0$ (or strategy WBA). After eliminating the strategy of choosing A at $t = 0$, as no one will choose A at $t = 0$, the game is essentially no different from our benchmark model where B is the only binding choice at $t = 0$. As there, WBA is the unique iteratively undominated strategy and, as a result, efficient coordination is achieved.

Finer Information

With finer information, for a player who chooses to wait, they can choose either A or B contingent on any distribution of the time 0 choices. Clearly, any strategies involving the choice of A after observing some B choices at $t = 0$ (i.e., $n(s_{-i}) = \big|\{j \in \mathcal{N} \setminus \{i\} | s_j = B\}\big| \geq 1$) or the choice of B after observing all others choosing A at $t = 0$ are weakly dominated. After elimination of these dominated strategies, there is a large set of surviving strategies due to the number of contingencies available for the choice at $t = 1$. Next, we resort to a three-player example of this game, and show that any remaining strategy survives iterated weak dominance. For example, each player may choose to wait and then take B on observing that at least one of the other players waited, and efficient coordination is not achieved.

A three-player example

Here, we consider a three-player example, and explore the strategy profiles that survive iterated weak dominance, as well as strategy profiles that constitute subgame-perfect Nash equilibria.

Let us classify all possible information sets (for a player who has waited) into four categories: (a) at least one other player chooses B at $t = 0$; (b) both of the other two players choose A at $t = 0$; (c) one other player waits and the remaining player chooses A at $t = 0$; and (d) both of the other two players wait.

Iterated Weak Dominance First, observe that all strategies that fit any of the following criteria are weakly dominated.

1. Any strategy that involves waiting and playing A at any information set of type (a) is weakly dominated by waiting and taking the same action at (b), (c), and (d), but playing B at (a).
2. Any strategy that involves waiting and playing B at (b) is weakly dominated by waiting and playing A at (b) and taking the same action at any other information set.
3. Choosing B at $t = 0$ is weakly dominated by waiting and choosing B except at (b).

Thus, any undominated strategy with waiting involves choosing B at (a), and choosing A at (b). With some abuse of notation, we can write the set of undominated strategies as $\mathcal{S}_u = \{A, wBA, wAB, wBB, wAA\}$. Here, A denotes the strategy of choosing A at $t = 0$. The strategy wBA involves waiting, choosing B at (c), choosing A at (d), choosing B at (a), and choosing A at (b). The other strategies in $\mathcal{S}_u$ are defined similarly.

We proceed to show that any strategy $s_u \in \mathcal{S}_u$ cannot be eliminated in the first round. Note that A cannot be dominated because it is the unique best response if the two other players both choose wAB. For any $s_u \in \mathcal{S}_u \setminus \{A\}$, s_u is the unique best response if one of the two other players chooses s_u, and the other chooses a mixed strategy $pA \oplus (1-p)s_u$ with $p \in (0, 1)$.

It is clear that these five strategies survive further rounds of elimination. Each of these strategies can be made a unique best response, so for none of them is there another strategy that generates the same payoffs in all contingencies. Therefore, ϵ-social preferences have no bite.

Subgame-Perfect Nash Equilibrium In the three-player game with full moritoring and in which both choices are binding at $t = 0$, it is easy to check that any strategy $\mathcal{S}_u = \{B, A, wBA, wBB, wAA\}$ constitutes a symmetric pure-strategy subgame-perfect equilibrium.

It is worth noting that the strategy profile $(s_i = wAB)_{i \in \mathcal{N}}$ does not constitute a subgame-perfect equilibrium, even though it survives iterated weak dominance. That is because, for any player, if other players choose wAB, then A is a strictly profitable deviation since choosing A at $t = 0$ causes other players to switch from taking B to taking A at $t = 1$.

Summary

When both A and B are binding choices at $t = 0$, the effectiveness of the delay option depends on the information structure. Using a 3-player example, we show that with finer information, multiple strategy profiles survive iterated weak dominance. Subgame-perfect equilibrium also fails to yield a unique prediction. Under either analysis, we cannot rule out the strategies that involve choosing B when some (or all) other players choose to wait at $t = 0$, and some other players choose A at $t = 0$. In any of these cases, efficient coordination cannot be achieved. This result should be easily extended to coordination game with the same irreversibility structure but played by more than three players. However, when players can only observe the binary message m, based on the unique iteratedly undominated strategy profile, the delay option leads to the efficient coordination.

1.3 Costly Delay

In the paper, we focus on a costless delay option and show that forward induction reasoning works under the assumption of ϵ social preferences. A player who holds the ϵ social preference would prefer taking B at $t = 0$ than waiting and taking B regardless of the message received. In fact, even without other-regarding preferences, this result and the operation of forward induction reasoning would hold if there is a small cost of delay. If delay is costly, the strategy WBB would yield a strictly lower payoff than the strategy B, regardless of other players' strategy profiles. Therefore, WBB is dominated by B in the presence of a delay cost.

However, when delay is costly, B is the unique best response when all other players choose B. Therefore, the strategy B survives iterated elimination of weakly dominated strategies. There is also an equilibrium in which all players choose B. As such, although the forward induction reasoning as well as the mechanism of signal intention via waiting can operate when delay is costly, the cost associated with the delay option may limit its effectiveness in promoting efficient coordination.

2 Experiments

2.1 Experimental Instructions ("BI-b")

(Translated into English from Chinese.)

Welcome to our experiment! This is an experiment on decision making. The following instructions will help you better understand this experiment so as to make good decisions and earn a greater amount of money. Your earnings in the experiment, together with a show-up fee of RMB 5, will be paid at the end of the session.

During the experiment, please do not talk or communicate with other participants. And please put away your phones. If you have any questions or need assistance of any kind, please raise your hand, and the experimenter will come to you. Otherwise, if you fail to obey these rules of the experiment, YOU WILL BE ASKED TO LEAVE. Thank you.

At the beginning of the experiment, all the participants will be randomly divided into groups of 4 people. You will participate in 15 rounds of decisions together with the other members of that group. The points you earn in each round will depend on the decisions that you and the other group members make. Your earnings will depend on the points accumulated in all 15 rounds, with an exchange rate of 100 Points = RMB 7. Your group members will not change throughout the 15 rounds, but their identity will never be disclosed.

Each of the group members will choose between two options, 1 and 2. The following table presents the relationship between decisions made by you and your group members, as well as the points that you earn in the round.

Your points in a round depend on your own choices, as well as on the minimum choice in your group.

		Minimum choice in your group: 1	Minimum choice in your group: 2
Your choice	choose 1	45	n.a.
	choose 2	5	55

If you choose 1, the minimum choice in your group must be 1. In this case, you will get 45 points. If you choose 2, there will be two possible outcomes:

1) If the smallest choice in your group is 1 (that is, at least one of your group members chooses 1), then you get 5 points.

2) If the smallest choice in your group is 2 (that is, none of your group members chooses 1), then you get 55 points.

In each round, your decision-making takes place in two stages.

Stage 1: each member in your group chooses between "1" and "wait." (The option "2" is not available in Stage 1.)

If you choose "1," it will be your final decision in this round.

If you choose "wait," you will then decide between "1" and "2" in Stage 2, based on the outcomes (to be discussed below) in Stage 1.

Stage 2: if you chose "wait" in stage 1, you now decide between "1" or "2" based on whether any group member chose "1" in Stage 1. To be specific, there are two possible outcomes from Stage 1:

Outcome 1: Some of the group members chose "1" in Stage 1. You have to decide, if this outcome occurs, whether to choose "1" or "2."

Outcome 2: None of the group members chose "1" in Stage 1. Again, you have to decide, if Outcome 2 occurs, whether to choose "1" or "2."

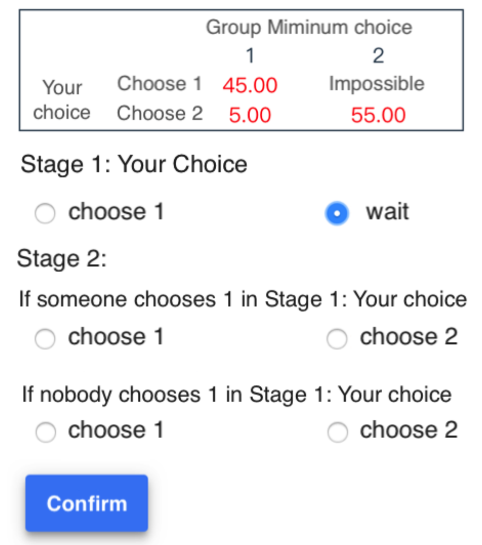

Figure 1: Screenshot of Choice Page (after choosing "Wait")

Note that the decisions in Stage 1 and Stage 2 will be made on the same page. Hence, you will not know the realized Stage 1 outcome when making decisions for Stage 2. Therefore, you have to make two decisions regarding the two possible outcomes (See Figure 1). Your first decision is your response to: "some of your group members chose "1" in Stage 1" (Outcome 1); and your second decision is your response to: "none of your group members chose "1" in Stage 1" (Outcome 2).

After all group members have finished their decisions for both stages and clicked "submit," the system will automatically determine whether any group member chose "1" in Stage 1. If some group members chose "1" (Outcome 1), your response to Outcome 1 will take effect, and it will be your final decision in this round. If none of your group members chose "1" in Stage 1 (Outcome 2), then your response to Outcome 2 will take effect, and it will be your final decision in this round.

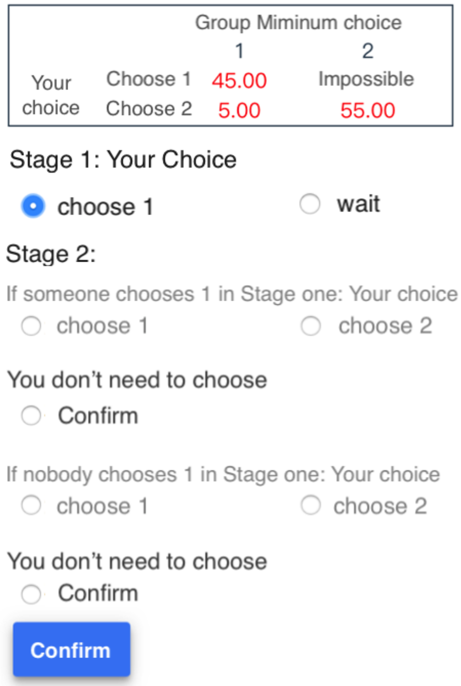

Figure 2: Screenshot of Choice Page (after Choosing "1")

If you chose "1" in Stage 1, you do not need to make any decision for Stage 2, but you will have to click the "Confirm" buttons (See Figure 2). As stated above, your final decision in this round will be "1."

At the end of each round, the interface will display (1) your choice in this round; (2) whether any group member chose "1" in Stage 1; (3) the minimum choice in this round; (4) the points you earned in this round; (5) the points you have accumulated in this round.

At the end of the experiment, the payments you will receive will depend on the total points earned (100 points = RMB 7, 1 point = RMB 0.07). You will also earn a RMB 5 show-up fee. You will be able to collect the payment after all participants in this session have finished.